بِسۡمِ اللّٰهِ الرَّحۡمٰنِ الرَّحِيۡمِ

"In the name of Allah ,the Most Beneficent ,the Most Merciful."

شروع کوم په نامه د الله(ج) چې رحمٰن او رحیم دی.

Narrated Abu Hurairah (RA): The Prophet (SAW) said, "Martyres are those who die because of drowning, plague, an abdominal disease, or of being buried alive by a falling building" (720-Al-Bukhari)

And then he added, "If the people knew (the reward for) the Zuhr prayer in its early time, they would race for it. If they knew (the reward for) the Isha and the Fajr prayers in congregation, they would join them even if they had to crawl. If they knew (the reward for) the first row, they would draw lots for it." (721-Al-Bukhari)

ګرانو او قدرمنو وروڼو اوخویندو!
السلام علیکم ورحمت الله وبرکاته!

له الله(ج) څخه هیله کوم چې تاسې ټول روغ او جوړ وئ.

دغه لاندې د ریاضي درسونه ما د انګلستان په یوه ښار کې چې سوتهمپټن نومېږي، د افغانانو اولادونو ته خاص د ثواب په خاطر درس ورکاوه. که چېرې دغه درسونه په صحیح توګه پکار واچول شي، نو ان شاءالله ځینې پلرونو او د دوی اولادونو ته به ګټور وي.

نوټ: دغه لړۍ به ان شاءالله، له لومړي ټولګي څخه تر پوهنتون پورې ان شاءالله، دوام ولري.

د یادونې وړ ده، چې زما مسلک ریاضي نده، او ما خپلې زده کړې، د دکتورا تر کچې د الکترانکس په برخه کې، په انګلستان کې سرته رسولې. همدارنګه، ما ۶ کاله د کابل پوهنتون، د انجینرۍ په پوهنځی، او ۲۷ کاله د انګلستان په یوه پوهنتون کې معلمي کړېده. یعني هغه چا چې په الکترانکس انجینرنګ کې زده کړې کړې وي، نو هغه د ریاضي په قدر پوهېږي. زه به له ریاضي درسونو وروسته، د الکترانکس په برخه کې به ان شاءالله درسونه شروع کړم.

Dear Brothers and Sisters! السلام علیکم و رحمت الله و برکاته

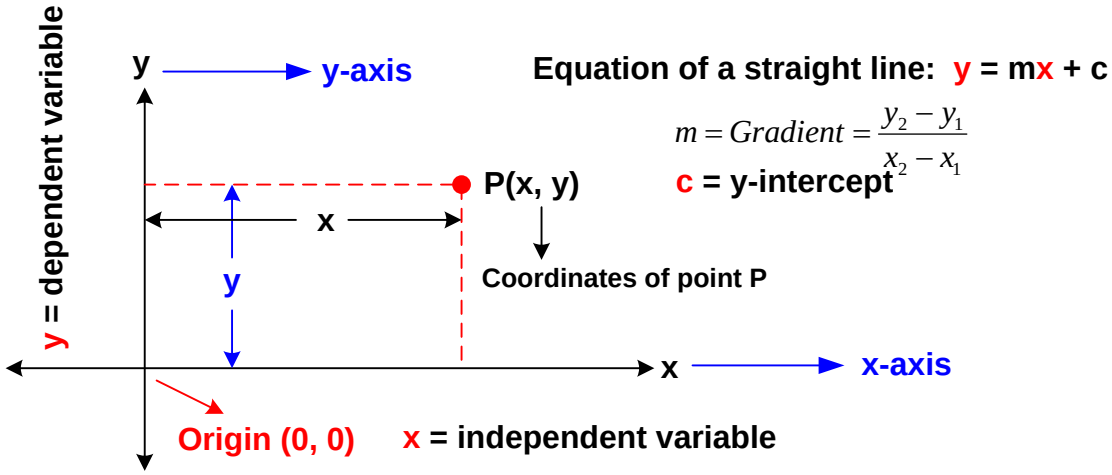
I kindly request each one of you to remember me, my parents, my children, my family and the entire Muslims of the world in your daily Prayers.

ګرانو او قدرمنو وروڼو اوخویندو!
السلام علیکم ورحمت الله وبرکاته!

له الله(ج) څخه هیله کوم چې تاسې ټول روغ او جوړ وئ، او الله(ج) د وکړي چې ماته ، زما مور او پلار ته ، زما اولادونو ته ، زما ټولې کورنۍ ته او د نړۍ ټولو مسلمانانو ته ، د زړه له کومې دعا وکړئ. والسلام

زما اړیکه : abdullahwardak53@gmail.com

Cartesian or Rectangular Coordinates System



Equation of a straight line: y = mx + c

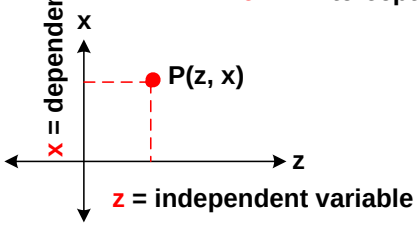
$$m = Gradient = \frac{y_2 - y_1}{x_2 - x_1}$$

c = y-intercept

Equation of a straight line: x = mz + c

$$m = Gradient = \frac{x_2 - x_1}{z_2 - z_1}$$

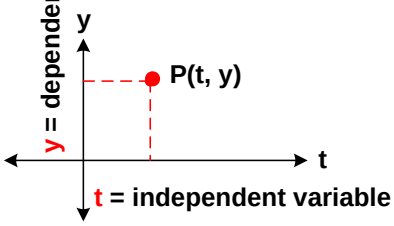
c = x-intercept



Equation of a straight line: y = mt + c

$$m = Gradient = \frac{y_2 - y_1}{t_2 - t_1}$$

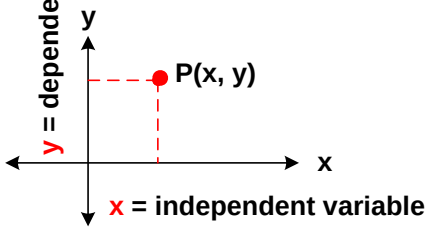
c = y-intercept



Equation of a straight line: y = mx + c

$$m = Gradient = \frac{y_2 - y_1}{x_2 - x_1}$$

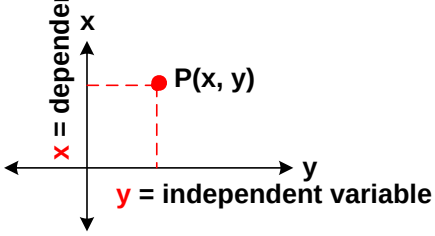
c = y-intercept



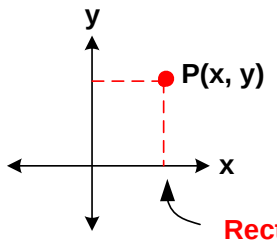
Equation of a straight line: x = my + c

$$m = Gradient = \frac{x_2 - x_1}{y_2 - y_1}$$

c = x-intercept



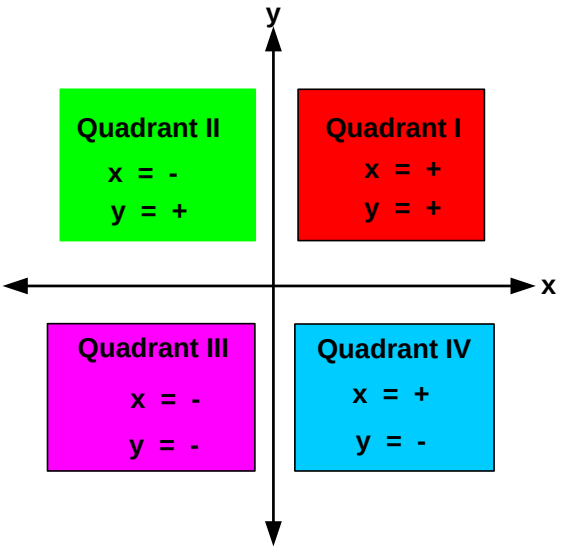
Note: However, the common one is: Rectangular or Cartesian coordinates system.

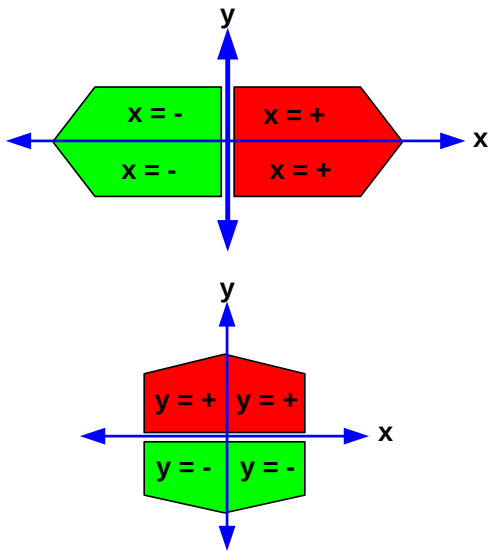


Equation of a straight line: $y = mx + c$

m is the gradient or slope and c is the y-intercept.

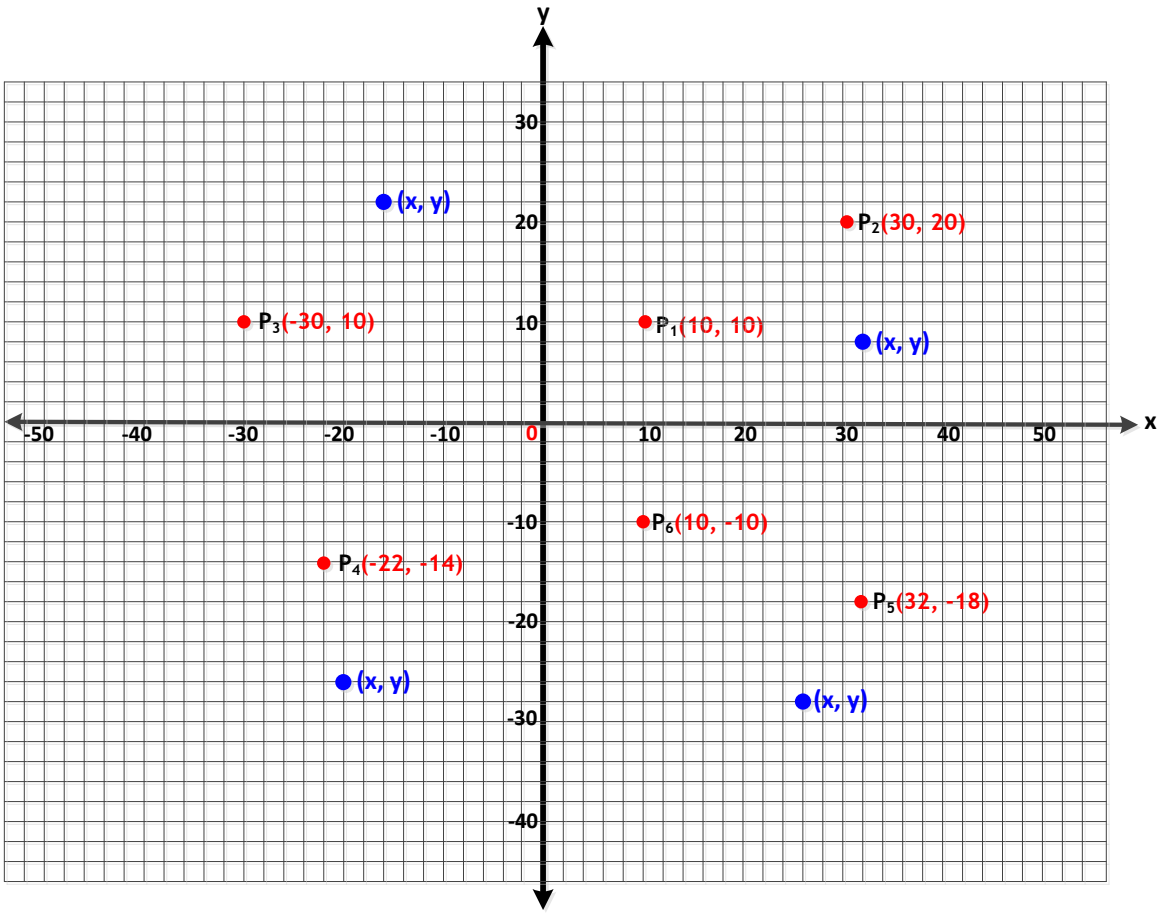
Rectangular or Cartesian coordinates system



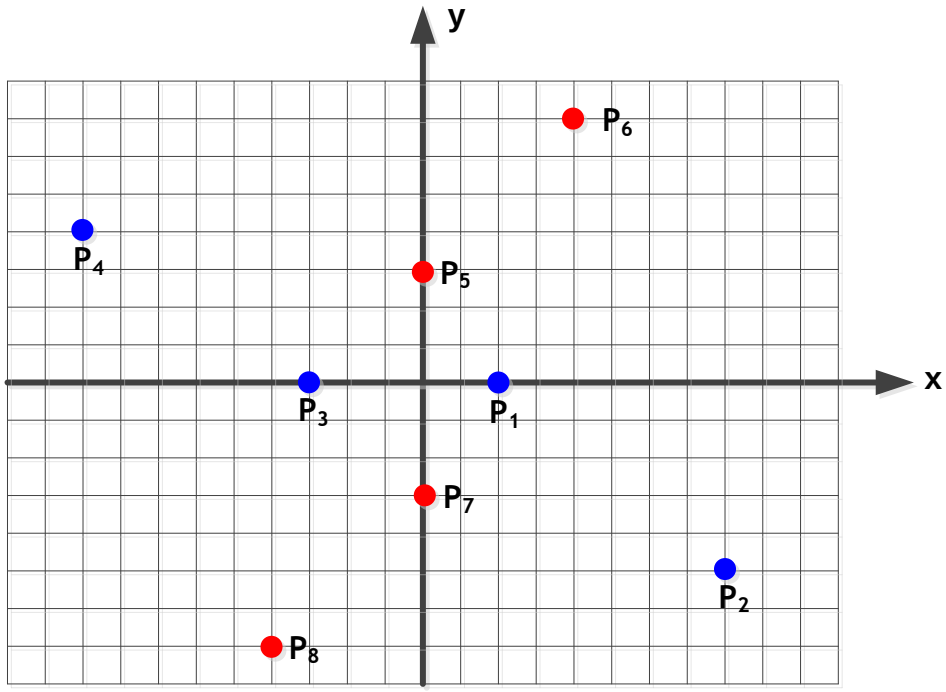


Example: Plot the following points on the graph below.

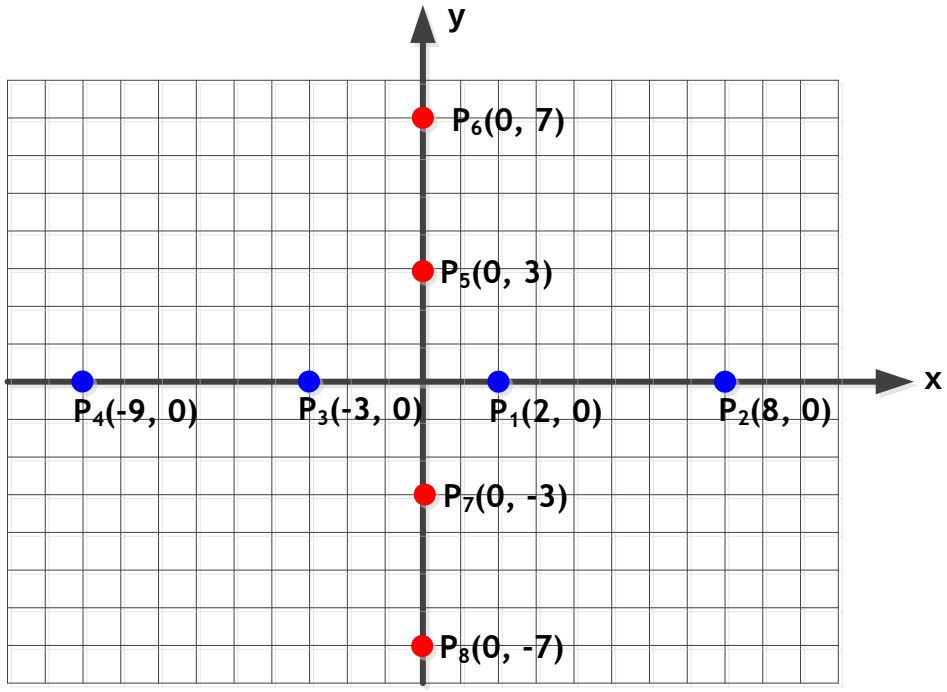
$P_1(10,10)$ $P_2(30,20)$ $P_3(-30,10)$ $P_4(-22,-14)$ $P_5(32,-18)$ $P_6(10,-10)$



Example: Write down the coordinates of the following points.



Ans:



$P_1(2, 0)$, $P_2(8, -5)$, $P_3(-3, 0)$, $P_4(-9, 4)$, $P_5(0, 3)$, $P_6(4, 7)$, $P_7(0, -3)$, $P_8(-4, -7)$

Plotting The Graphs

Examples: **Plot the graphs of the following straight lines.**

(a) $y = x + 4$ (b) $y = 4 - \frac{2}{3}x$ (c) $y = 3x - 6$ (d) $y = -3x + 6$

Ans:

(a)

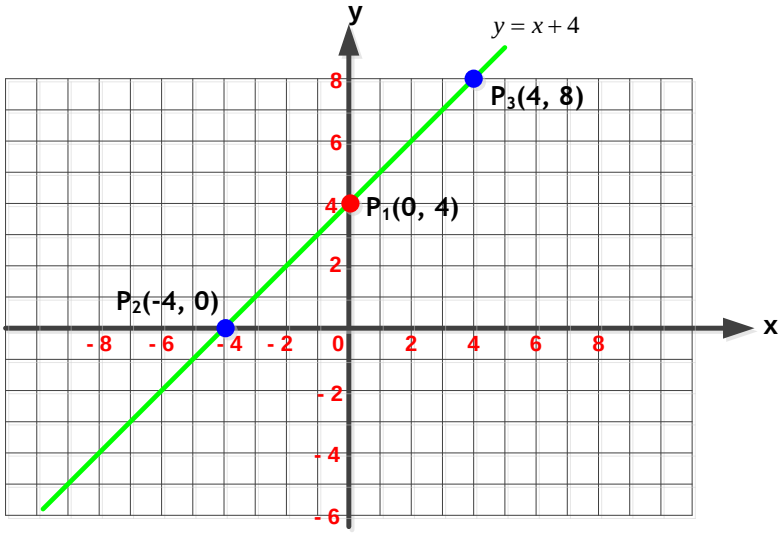
$y = x + 4$

x	y
0	4
- 4	0

OR

x	y
0	4
4	8

$P_1(0, 4)$ **and** $P_2(-4, 0)$ **OR** $P_1(0, 4)$ **and** $P_3(4, 8)$



(b)

$y = 4 - \frac{2}{3}x$

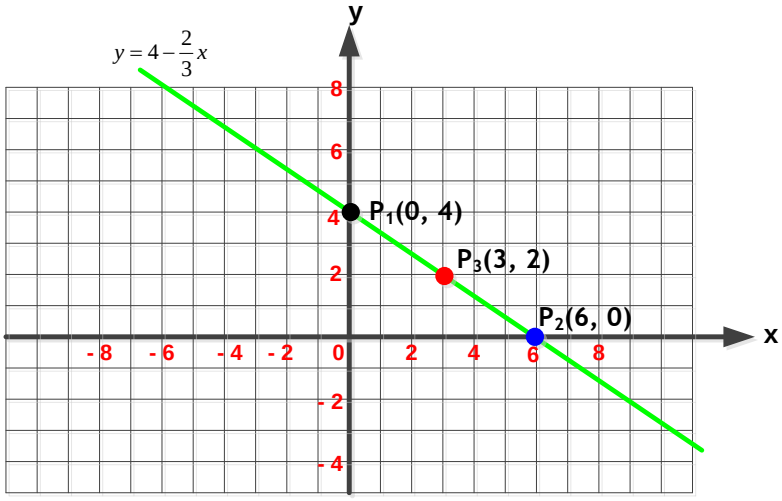
$y = -\frac{2}{3}x + 4$

x	y
0	4
6	0

OR

x	y
0	4
3	2

$P_1(0, 4)$ **and** $P_2(6, 0)$ **OR** $P_1(0, 4)$ **and** $P_3(3, 2)$



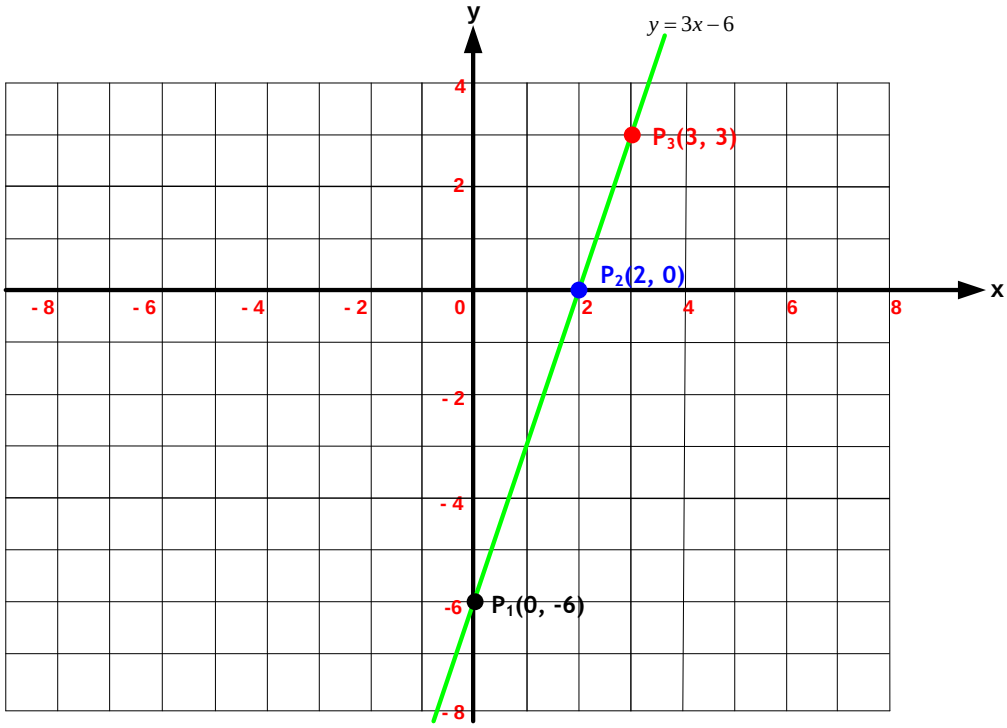
(c) $y = 3x - 6$

x	y
0	-6
2	0

OR

x	y
0	-6
3	3

$P_1(0, -6)$ **and** $P_2(2, 0)$ **OR** $P_1(0, -6)$ **and** $P_3(3, 3)$



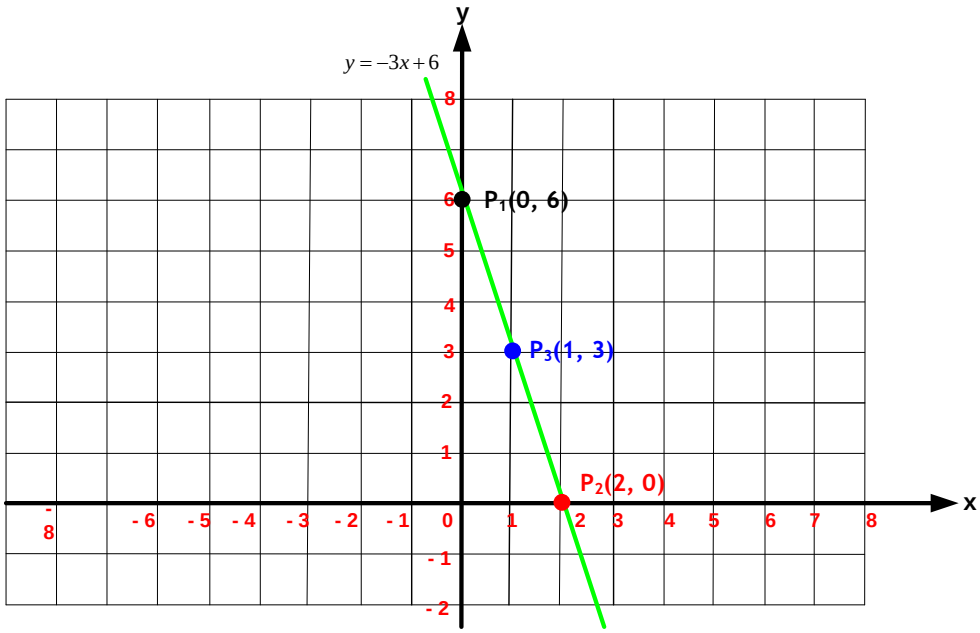
(d) $y = -3x + 6$

x	y
0	6
2	0

OR

x	y
0	6
1	3

$P_1(0, 6)$ **and** $P_2(2, 0)$ **OR** $P_1(0, 6)$ **and** $P_3(1, 3)$

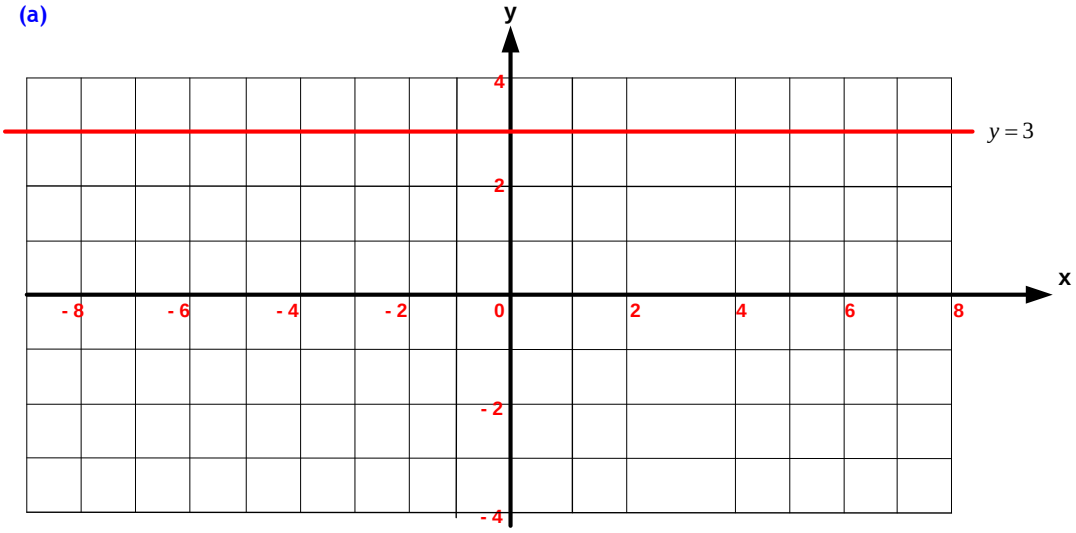


Examples: Plot the graphs of the following straight lines.

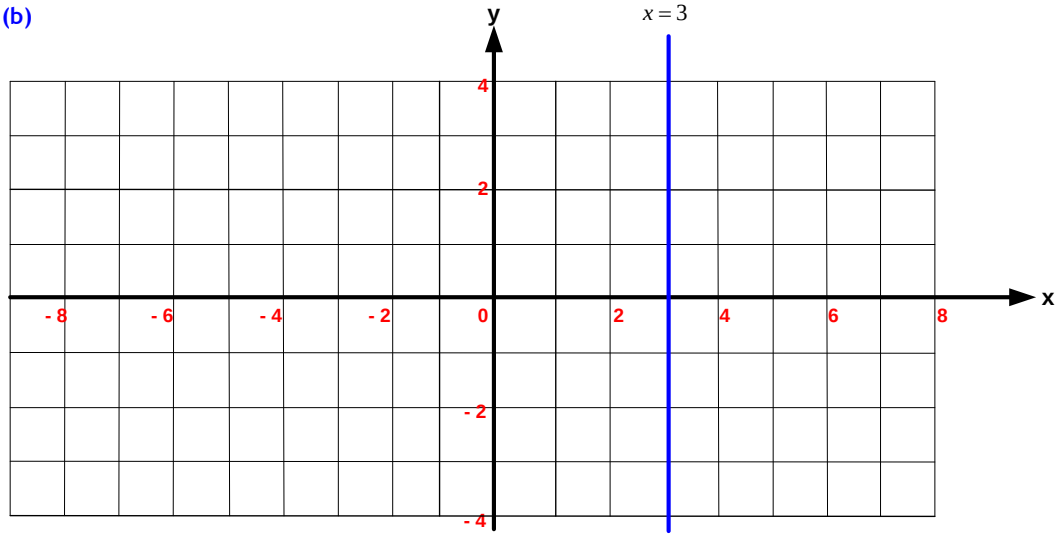
(a) $y = 3$ (b) $x = 3$ (c) $y = -3$

Ans:

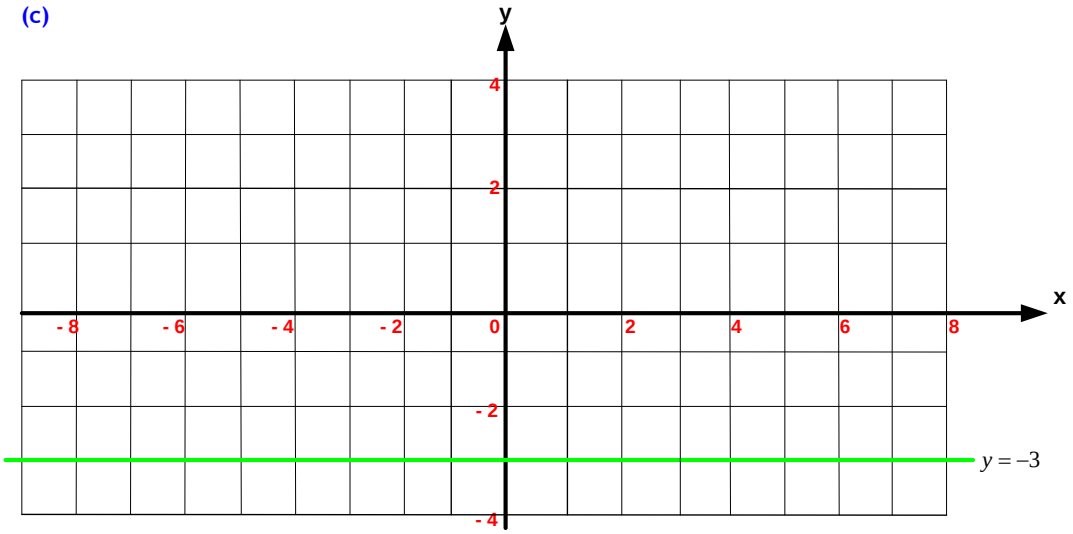
(a)



(b)



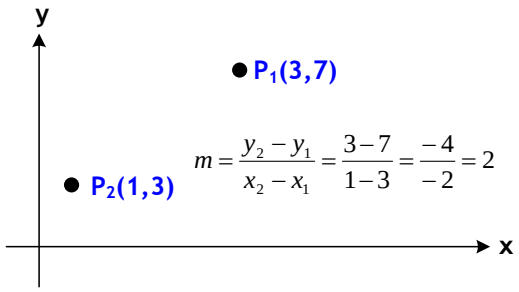
(c)



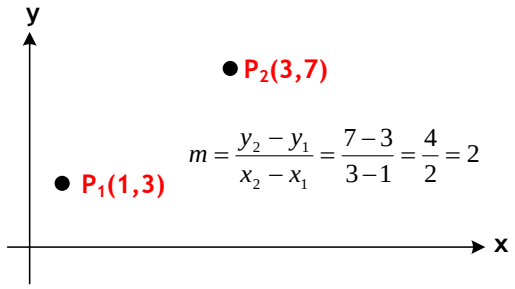
Equation of a straight line

The **standard form** of a straight line equation is of the form: $y = mx + c$
Where m is the **gradient** (**slope**) and c is the **y-intercept.**

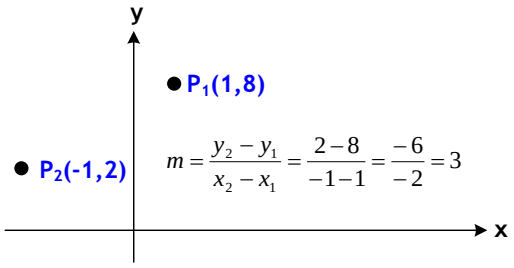
The **gradient** $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and it does not matter which point is taken as P_1 and P_2 as it can be seen as follow:



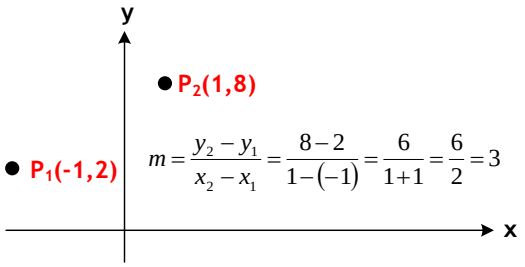
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-7}{1-3} = \frac{-4}{-2} = 2$$



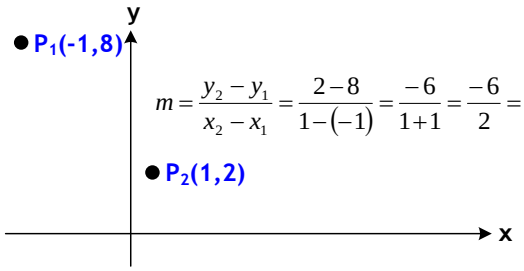
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{3-1} = \frac{4}{2} = 2$$



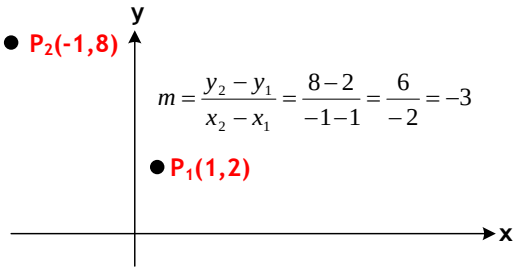
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-8}{-1-1} = \frac{-6}{-2} = 3$$



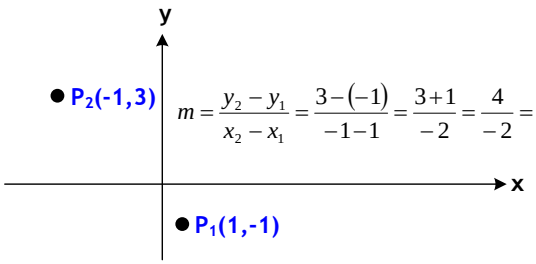
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{1-(-1)} = \frac{6}{1+1} = \frac{6}{2} = 3$$



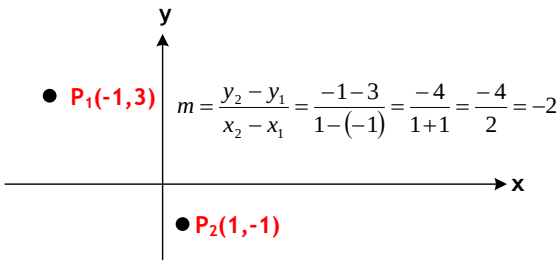
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-8}{1-(-1)} = \frac{-6}{1+1} = \frac{-6}{2} = -3$$



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{-1-1} = \frac{6}{-2} = -3$$

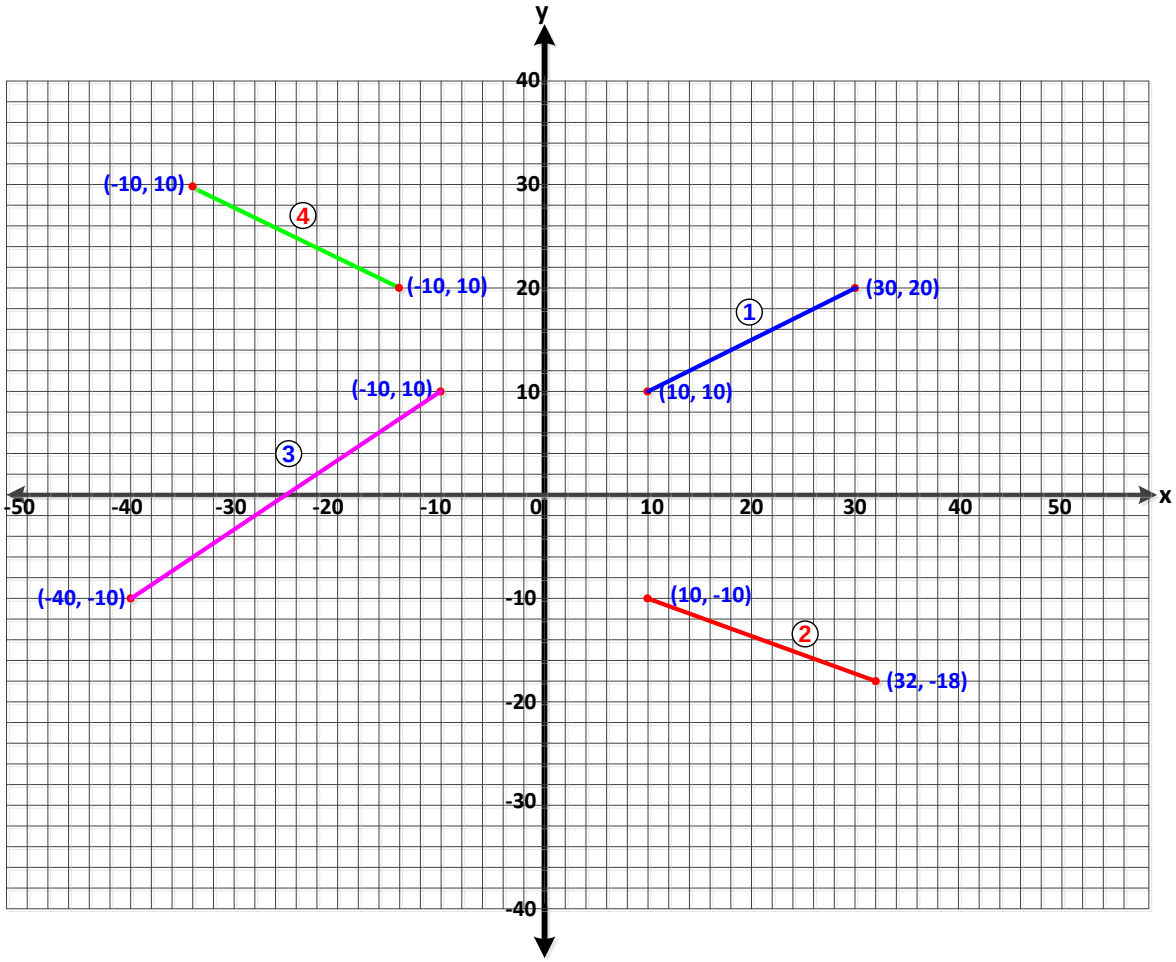


$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-1)}{-1-1} = \frac{3+1}{-2} = \frac{4}{-2} = -2$$



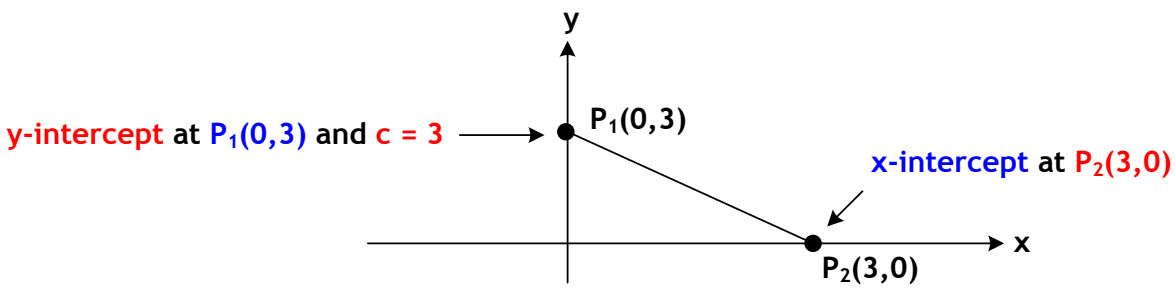
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{1-(-1)} = \frac{-4}{1+1} = \frac{-4}{2} = -2$$

Positive and Negative Gradients



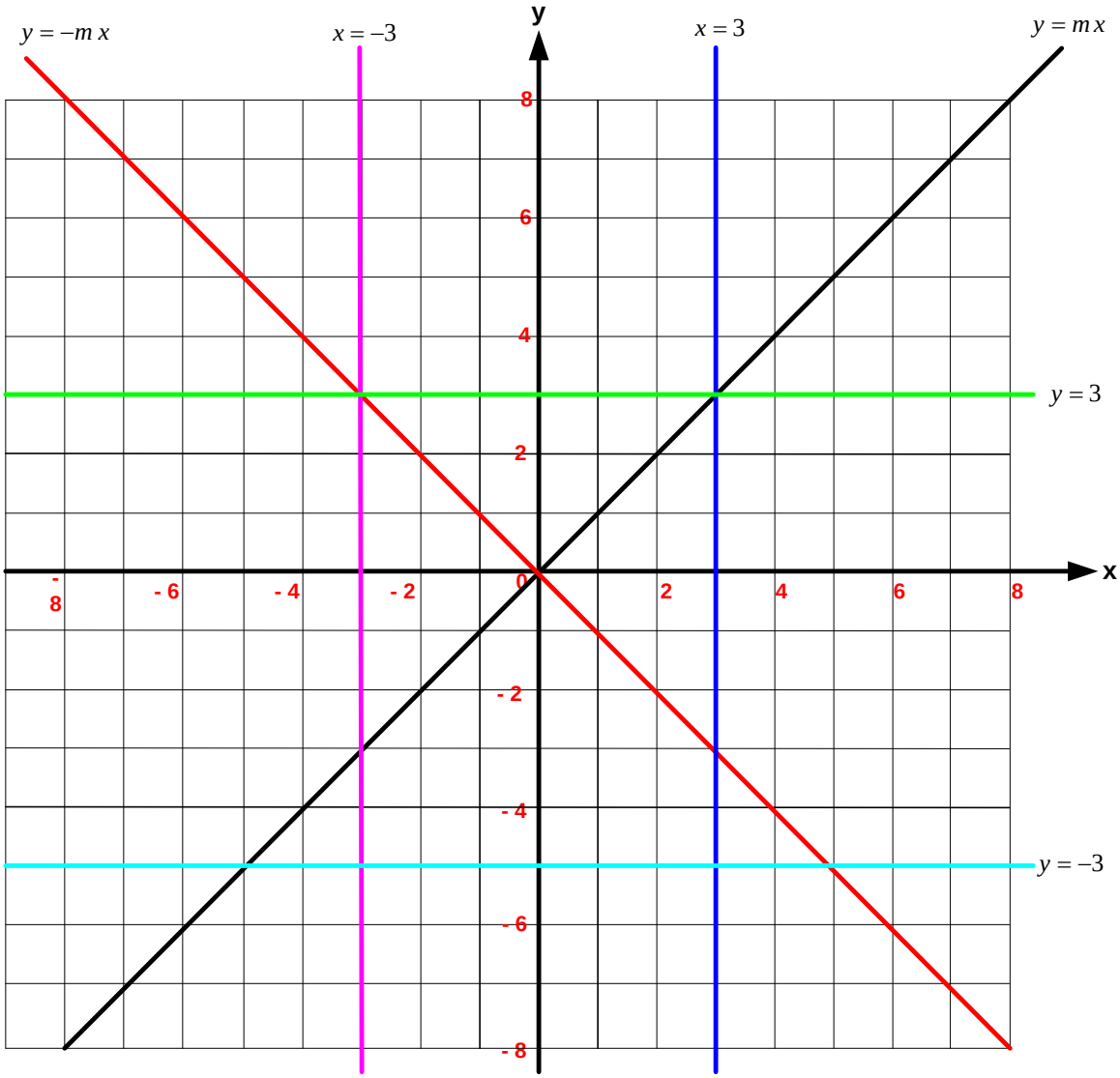
Note: The gradients of lines 1 and 3 are positive and the gradients of lines 2 and 4 are negatives.

Y-Intercept and X-Intercept



Consider the following graph. There are equations for various straight lines. The equations of these straight line are as follows:

1. $y = mx$ (Black Colour)
2. $y = -mx$ (Red Colour)
3. $y = 3$ (Green Colour)
4. $y = -3$ (Light Blue Colour)
5. $x = 3$ (Blue Colour)
6. $x = -3$ (Pink Colour)



Example: Find the equation of a straight line which passes through points P_1 and P_2 as shown in Fig. 1

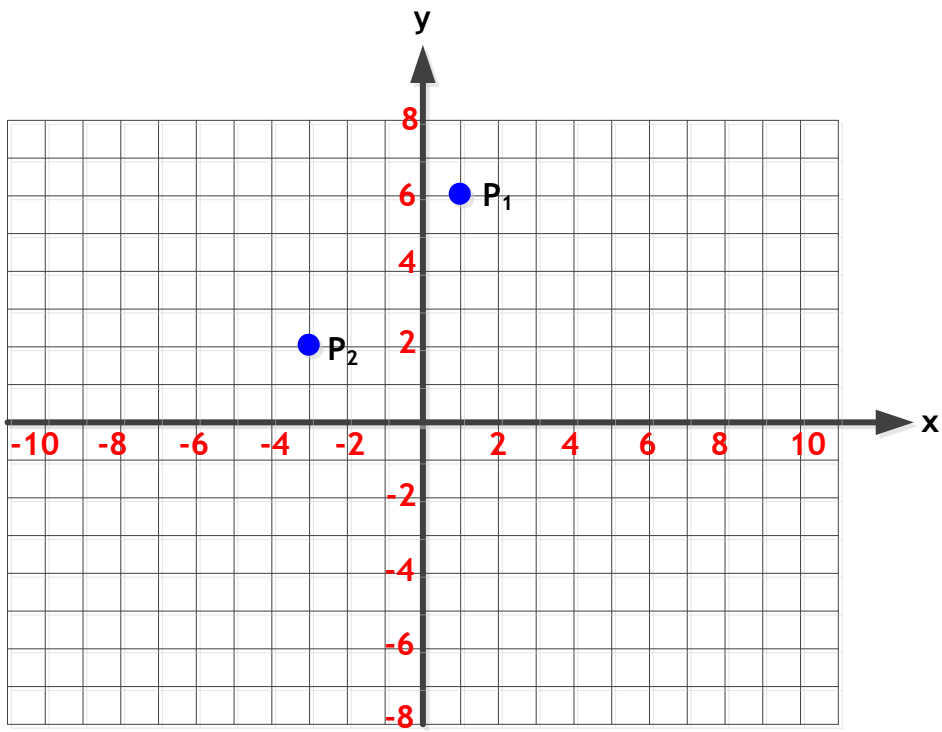


Fig. 1

Ans:

The standard equation of a straight line is: $y = mx + c$, so we need to find the **gradient**, m and the **y-intercept** c as follows:

$P_1(1,6)$ and $P_2(-3,2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-6}{-3-1} = \frac{-4}{-4} = 1$$

$$y = mx + b$$

$$y = x + c$$

$$6 = 1 + c$$

$$c = 6 - 1$$

$$c = 5$$

$$y = x + 5$$

Example: Find the equation of a straight line as shown in Fig.2

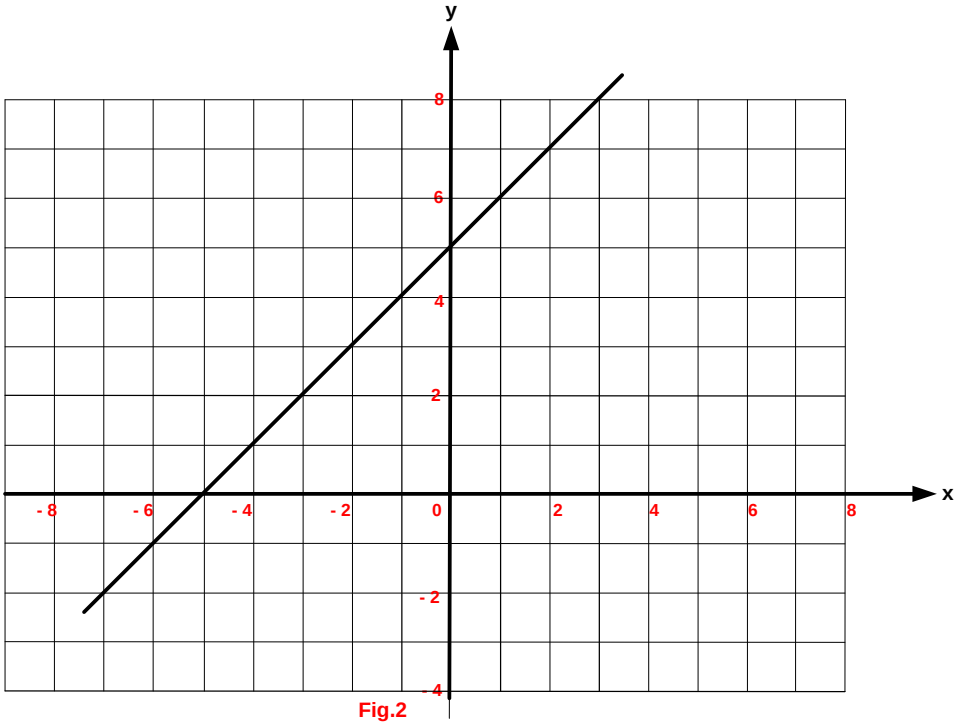


Fig.2

Ans:

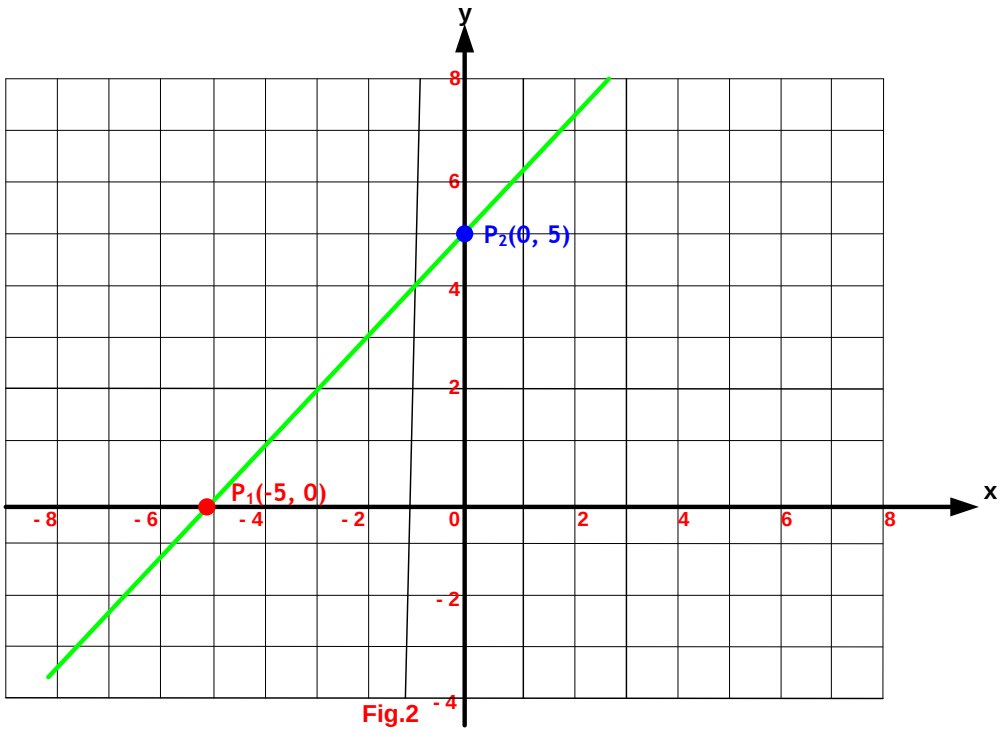


Fig.2

Looking at the graph, the line crosses the y axis at (0, 5), and therefore, the y-intercept ***c*** = 5. The gradient is positive and it is: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-0}{0-(-5)} = \frac{5}{5} = 1$

$y = mx + c$

$y = x + 5$

Example: Find the equation of a straight line as shown in Fig.3

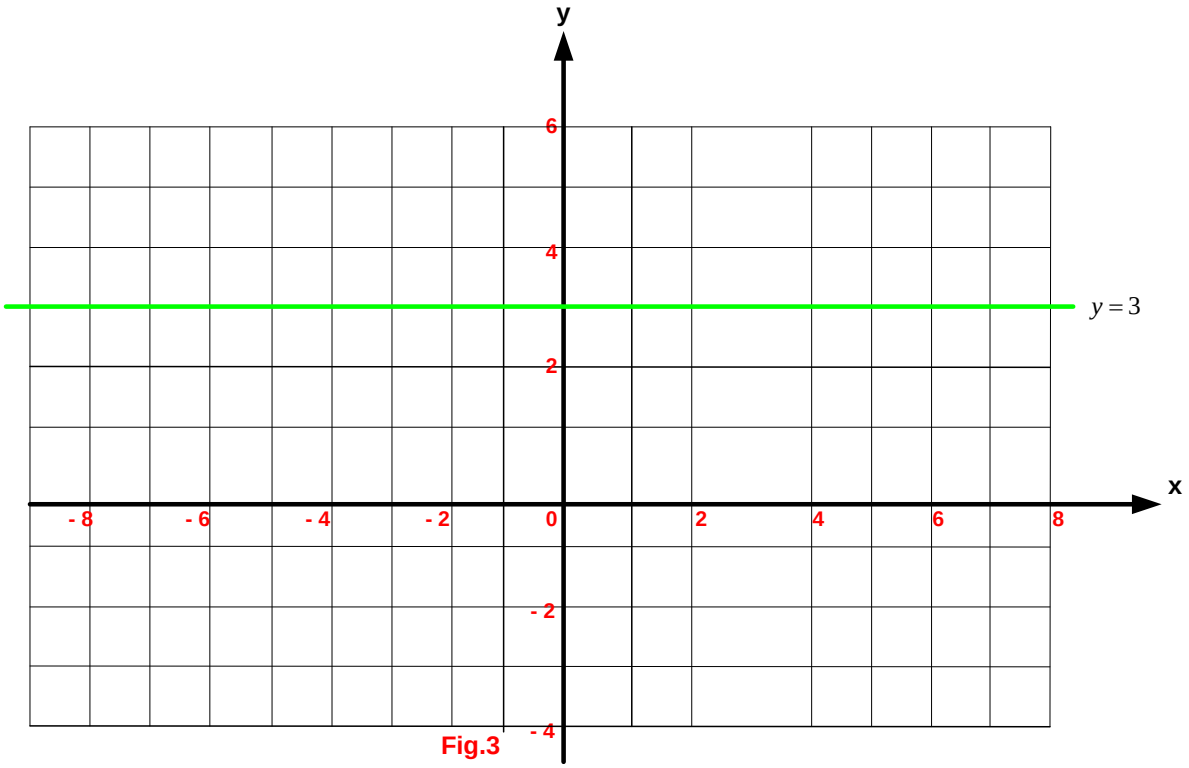


Fig.3

Ans: $y = 3$

Example: Find the equation of a straight line as shown in Fig.4

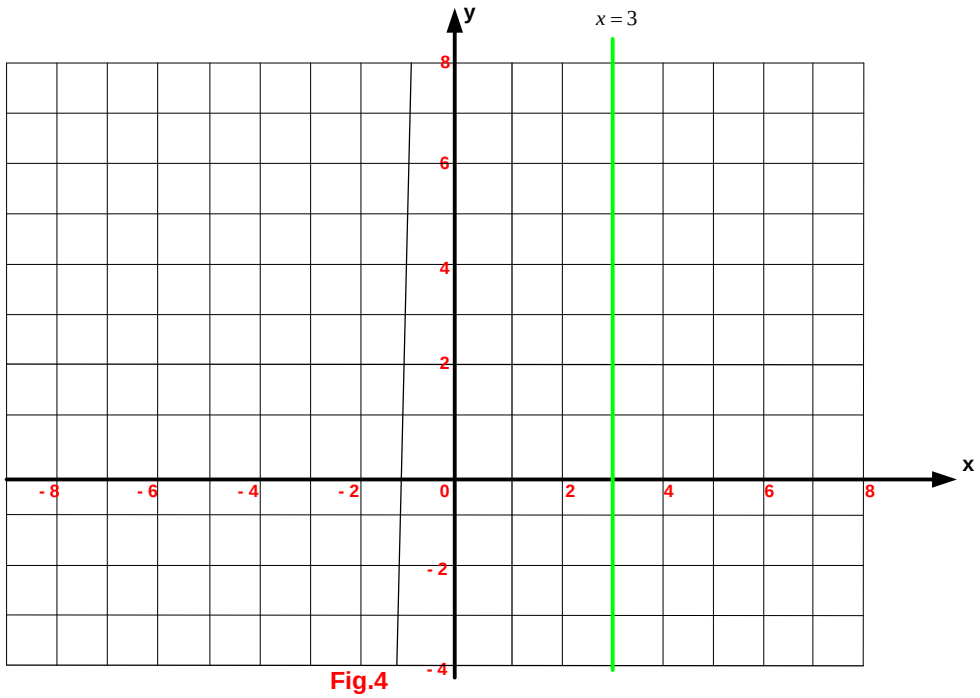


Fig.4

Ans: $x = 3$

Example: **Find the equation of a straight line as shown in Fig.5**

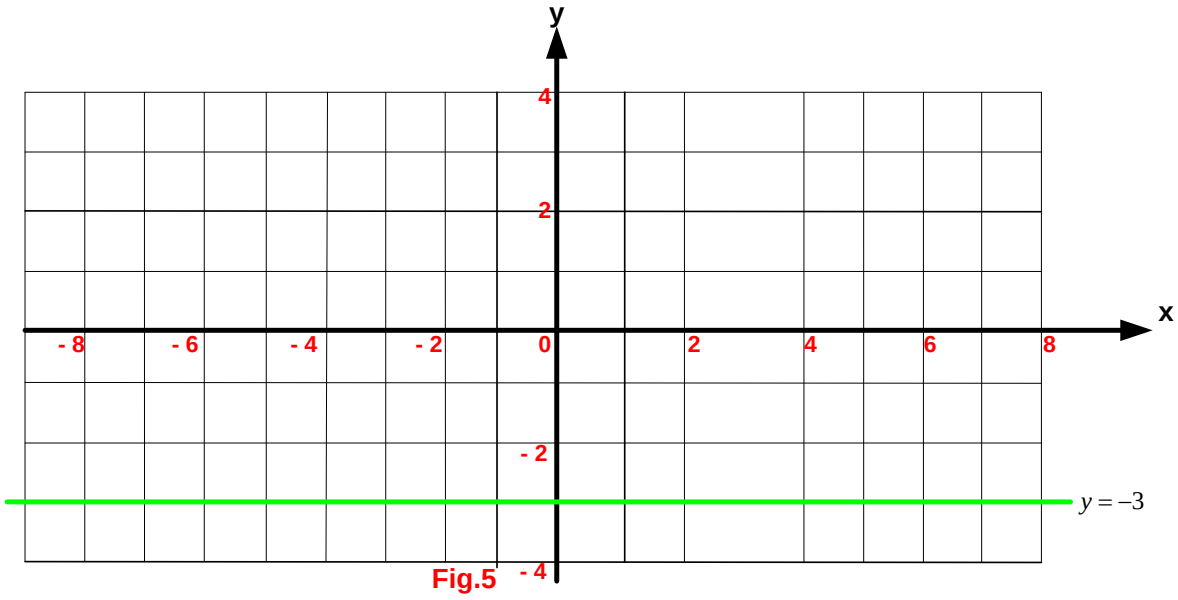


Fig.5

Ans: $y=-3$

Example: **Copy and complete the table for each of these equations.**

Equation	Gradient	Coordinates of y-intercept
$y=3x+2$		
$y=4-3x$		
$2y=-3x+7$		
$\frac{1}{3}y=4-\frac{3}{2}x$		

(a) $y=3x+2$ (b) $y=4-3x$

(c) $2y=-3x+7$ (d) $\frac{1}{3}y=4-\frac{3}{2}x$

Ans:

(a) $y=3x+2$

$y=mx+c$ Equation of a straight line

Hence, $m=gradient=3$ and $y-\text{intercept}=c=2$

(b) $y=4-3x$

$y=-3x+4$

$y=mx+c$ Equation of a straight line

Hence, $m=gradient=-3$ and $y-\text{intercept}=c=4$

(c) $2y = -3x + 7$

$y = -\frac{3}{2}x + \frac{7}{2}$

$y = mx + c$ Equation of a straight line

Hence, $m = gradient = -\frac{3}{2}$ and $y - intercept = c = \frac{7}{2}$

(d) $\frac{1}{3}y = 4 - \frac{3}{2}x$

$y = -\frac{9}{2}x + 12$

$y = mx + c$ Equation of a straight line

Hence, $m = gradient = -\frac{9}{2}$ and $y - intercept = c = 12$

Equation	Gradient	Coordinates of y-intercept
$y = 3x + 2$	3	$(0, 2)$
$y = 4 - 3x$	-3	$(0, 4)$
$2y = -3x + 7$	$-\frac{3}{2}$	$\left(0, \frac{7}{2}\right)$
$\frac{1}{3}y = 4 - \frac{3}{2}x$	$-\frac{9}{2}$	$(0, 12)$

Example: Write the equations of these **FIVE** straight lines.

(a) Line **A** has a gradient of **2** and y-intercept at **(0, 1).**

Ans: $y = mx + c$

$m = 2$

$c = 1$

$y = 2x + 1$

(b) Line **B** cuts the y-axis at **-4** and has a gradient of **1**.

Ans: $y = mx + c$

$m = 1$

$c = -4$

$y = x - 4$

(c) Line **C** has **m = -3** and $c = \frac{1}{2}$

Ans: $y = mx + c$

$m = -3$

$c = \frac{1}{2} \Rightarrow y = -3x + \frac{1}{2}$

(d) Line **D** cuts the x-axis at **-4** and has gradient of **2**.

Ans:
$$y = mx + c$$
$$y = 2x + c$$
$$0 = 2\times(-4) + c$$
$$c = 8$$
$$y = 2x + 8$$

(e) Line **E** passes through the origin and has gradient of **1**.

Ans:
$$y = mx$$
$$y = x$$

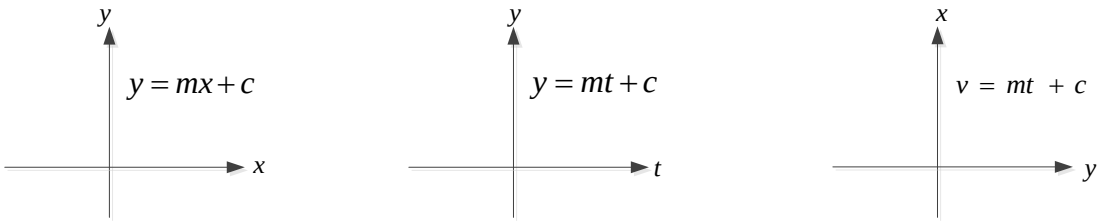
Example: Find the equation of the line that passes through the points **(-3,5)** and **(5,-7).**

Ans:

First, find the slope: Where $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-7)}{-3-(-5)} = \frac{5+7}{-3+5} = \frac{12}{2} = 6$

$$y = mx + c = 6x + c$$
$$5 = 6\times(-3) + c \Rightarrow c = 5 + 18 = 23$$
$\Rightarrow y = 6x + 23$ is the equation of the line.

Important Information



$y = mx + c$: Gradient or Slope; y-intercept

$y = mt + c$: Gradient or Slope; y-intercept

$x = my + c$: Gradient or Slope; y-intercept

Example: Find the gradient and the y-intercept of $y = 3x + 7$.

Ans: **Gradient** $= m = 3$ **and** **y-intercept** $= c = 7$

Example: Find the gradient and the **y-intercept** of $-y = 3x + 7$.

Ans: $-y = 3x + 7 \quad \Rightarrow y = -3x - 7$

Gradient $= m = -3$ **and** **y-intercept** $= c = -7$

Example: Find the gradient and the y-intercept of $2y = 3x + 7$.

Ans:

$$2y = 3x + 7$$
$$y = \frac{3}{2}x + \frac{7}{2}$$

Gradient $= m = \frac{3}{2}$ and **y-intercept** $= c = \frac{7}{2}$

Example: Find the gradient and the y-intercept of $2y - 3x = 7$.

Ans:

$$2y - 3x = 7$$
$$2y = 3x + 7$$
$$y = \frac{3}{2}x + \frac{7}{2}$$

Gradient $= m = \frac{3}{2}$ and **y-intercept** $= c = \frac{7}{2}$

Example: Find the gradient and the y-intercept of $\frac{1}{2}y - 3x - 7 = 0$.

Ans:

$$\frac{1}{2}y - 3x - 7 = 0$$
$$\frac{1}{2}y = 3x + 7$$
$$y = 2(3x + 7)$$
$$y = 6x + 14$$

Gradient $= m = 6$ and **y-intercept** $= c = 14$

Example: Find the gradient and the y-intercept of $-3x - 7 = -2y$.

Ans:

$$-3x - 7 = -2y$$
$$2y = 3x + 7$$
$$y = \frac{3}{2}x + \frac{7}{2}$$

Gradient $= m = \frac{3}{2}$ and **y-intercept** $= c = \frac{7}{2}$

Example: Find the gradient and the y-intercept of $\frac{3}{5}y - \frac{3}{4}x - 7 = 0$.

Ans:

$$\frac{3}{5}y - \frac{3}{4}x - 7 = 0$$
$$\frac{3}{5}y = \frac{3}{4}x + 7$$
$$\frac{5}{3}\left(\frac{3}{5}y\right) = \left(\frac{5}{3}\right)\left(\frac{3}{4}x + 7\right)$$
$$y = \frac{15}{12}x + \frac{35}{3}$$

Gradient $= m = \frac{15}{12}$ and **y-intercept** $= c = \frac{35}{3}$

Example: Find the gradient and the y-intercept of $-\frac{2}{5}y+\frac{3}{7}x-7=0$.

Ans:

$$-\frac{2}{5}y+\frac{3}{7}x-7=0$$
$$-\frac{2}{5}y=-\frac{3}{7}x+7$$
$$\frac{5}{2}\left(-\frac{2}{5}y\right)=\left(\frac{5}{2}\right)\left(-\frac{3}{7}x+7\right)$$
$$-y=-\frac{15}{14}x+\frac{35}{2}$$
$$y=\frac{15}{14}x-\frac{35}{2}$$

Gradient $=m=\frac{15}{14}$ and **y-intercept** $=c=-\frac{35}{2}$

Example: Find the equation of a line with gradient of **3** and y-intercept of **-5**.

Ans:

$$y=mx+c$$
$$y=3x-5$$

Example: Find the equation of a line with gradient of **3** and passes through point **A(0, 5)** .

Ans:

$$y=mx+c$$
$$y=3x+c$$
$$5=3\times0+c$$
$$c=5$$
$$y=3x+5$$

Example: Find the equation of a line which passes through points **A(0, 5)** and **B(2, 11).**

Ans:

$$y=mx+c$$
$$m=\frac{y_2-y_2}{x_2-x_1}=\frac{11-5}{2-0}=\frac{6}{2}=3$$
$$y=3x+c$$
$$5=3\times0+c$$
$$c=5 \quad \Rightarrow y=3x+5$$

Example: Find the equation of a line which passes through points **A(-2, -13)** and **B(1, -4).**

Ans:

$$y=mx+c$$
$$m=\frac{y_2-y_2}{x_2-x_1}=\frac{-4-(-13)}{1-(-2)}=\frac{-4+13}{1+2}=\frac{9}{3}=3$$
$$y=3x+c$$
$$-4=3\times1+c \quad \Rightarrow c=-4-3=-7 \qquad \Rightarrow y=3x-7$$

Example: **Find the gradient and the y-intercept of the following straight lines.**

(a) $y=-4x+17$

Ans: $y=mx+c$

$\Rightarrow m=-4 \quad and \quad c=17$

(b) $2y=-4x+16$

Ans: $y=-\frac{4}{2}x+\frac{16}{2}$

$y=-2x+8$

$\Rightarrow m=-2 \quad and \quad c=8$

(c) $3y+4x=12$

Ans: $y=mx+c$

$3y=-4x+12$

$y=-\frac{4}{3}x+\frac{12}{3}$

$y=-\frac{4}{3}x+4$

$\Rightarrow m=-\frac{4}{3} \quad and \quad c=4$

(d) $\frac{x}{4}+\frac{y}{3}=1$

Ans: $y=mx+c$

$\frac{3x+4y}{12}=1$

$3x+4y=12$

$4y=-3x+12$

$y=-\frac{3}{4}x+3$

$\Rightarrow m=-\frac{3}{4} \quad and \quad c=3$

(e) $\frac{x}{4}-\frac{y}{3}=1$

Ans: $y=mx+c$

$\frac{3x-4y}{12}=1$

$3x-4y=12$

$-4y=-3x+12$

$y=\frac{-3}{-4}x+\frac{12}{-4}$

$y=\frac{3}{4}x-3 \qquad \Rightarrow m=\frac{3}{4} \quad and \quad c=-3$

(f) $\frac{x}{4}-\frac{y}{3}=2$

Ans: $y=mx+c$

$$\frac{3x-4y}{12}=2$$
$$3x-4y=24$$
$$-4y=-3x+24$$
$$y=\frac{-3}{-4}x+\frac{24}{-4}$$
$$y=\frac{3}{4}x-6$$
$$\Rightarrow m=\frac{3}{4} \quad and \quad c=-6$$

(g) $\frac{x}{a}+\frac{y}{b}=1$ where **a** and **b** are real numbers and **not zeros**.

Ans: $y=mx+c$

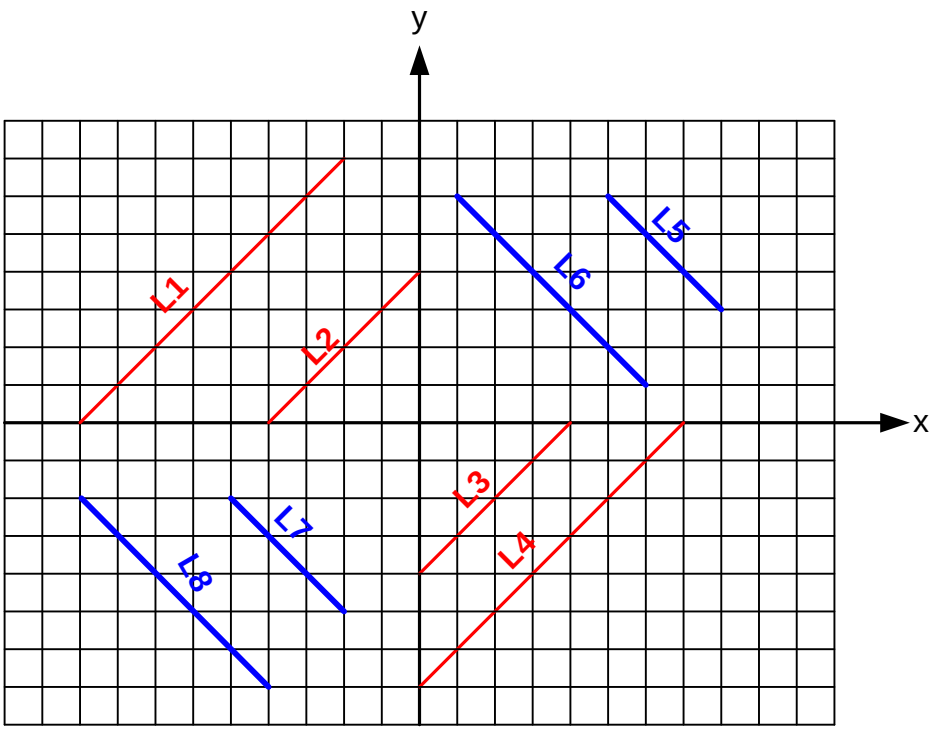
$$\frac{bx+ay}{ab}=1$$
$$bx+ay=ab$$
$$ay=-bx+ab$$
$$y=\frac{-bx}{a}+\frac{ab}{a}$$
$$y=-\frac{b}{a}x+b$$
$$\Rightarrow m=-\frac{b}{a} \quad and \quad c=b$$

(h) $5x+8y=3$

Ans: $y=mx+c$

$$5x+8y=3$$
$$8y=3-5x$$
$$8y=-5x+3$$
$$y=-\frac{5}{8}x+\frac{3}{8}$$
$$\Rightarrow m=-\frac{5}{8} \quad and \quad c=\frac{3}{8}$$

Gradients of Parallel Lines



Gradient of L_1: $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-0}{-2-(-9)} = \frac{7}{-2+9} = \frac{7}{7} = 1$

Gradient of L_2: $m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{0-(-4)} = \frac{4}{4} = 1$

Gradient of L_3: $m_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-4)}{4-0} = \frac{4}{4} = 1$

Gradient of L_4: $m_4 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-7)}{7-0} = \frac{7}{7} = 1$

Gradient of L_5: $m_5 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-6}{8-5} = \frac{-3}{3} = -1$

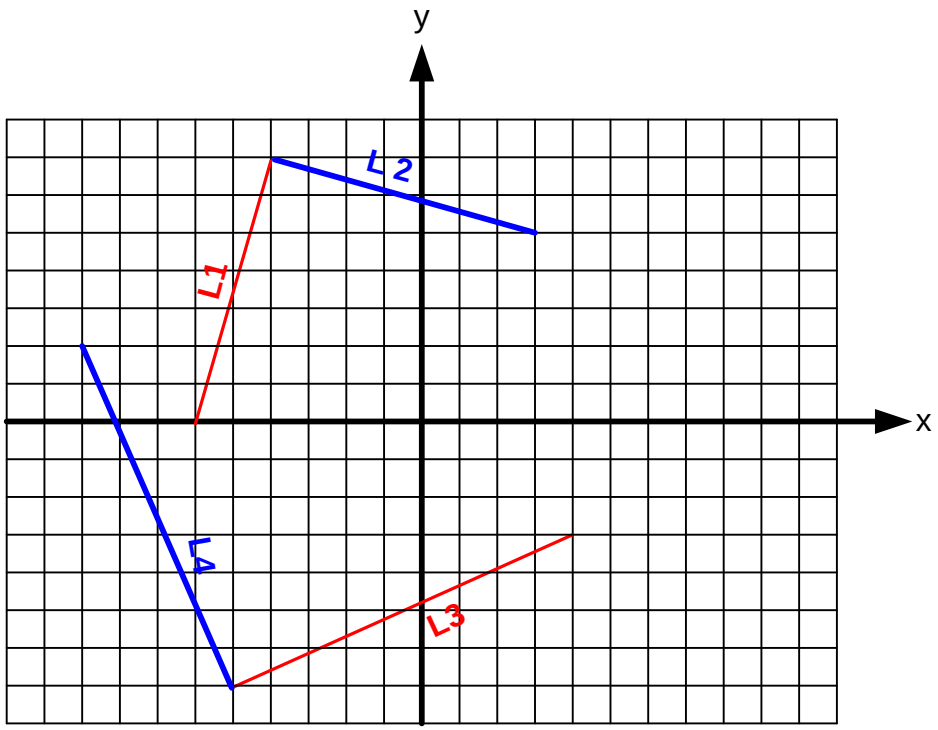
Gradient of L_6: $m_6 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-6}{6-1} = \frac{-5}{5} = -1$

Gradient of L_7: $m_7 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-5)}{-5-(-2)} = \frac{-2+5}{-5+2} = \frac{3}{-3} = -1$

Gradient of L_8: $m_8 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-7)}{-9-(-4)} = \frac{-2+7}{-9+4} = \frac{5}{-5} = -1$

Note: When two straight lines are **parallel**, then their **gradients** are the same. For example, lines L_1, L_2, L_3, L_4 are parallel with each other, then their gradients: $m_1 = m_2 = m_3 = m_4 = 1$. Also, lines L_5, L_6, L_7, L_8 are parallel with each other, then their gradients: $m_5 = m_6 = m_7 = m_8 = -1$.

Gradients of Perpendicular Lines



Gradient of L_1: $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-0}{-4-(-6)} = \frac{7}{-4+6} = \frac{7}{2}$

Gradient of L_2: $m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-5}{-4-3} = \frac{2}{-7} = -\frac{2}{7}$

Gradient of L_3: $m_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-(-7)}{4-(-5)} = \frac{-3+7}{4+5} = \frac{4}{9}$

Gradient of L_4: $m_4 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-2}{-5-(-9)} = \frac{-9}{-5+9} = \frac{-9}{4} = -\frac{9}{4}$

Note: When two straight lines are **perpendicular**, then their **gradients** are: $m_1 = -\frac{1}{m_2}$, **or** $m_2 = -\frac{1}{m_1}$, **or** $m_2 m_1 = -1$. For example, lines L_1 and L_2 are perpendicular with each other, then their gradients: $m_1 = \frac{7}{2}$ and $m_2 = -\frac{2}{7}$, *i.e.* $m_1 = -\frac{1}{m_2}$. Also, lines L_3 and L_4 are perpendicular with each other, then their **gradients**: $m_3 = \frac{4}{9}$ and $m_4 = -\frac{9}{4}$, *i.e.* $m_3 = -\frac{1}{m_4}$, **or** $m_1 m_2 = \frac{7}{2} \times \left(-\frac{2}{7}\right) = -1$

Example: Find the equation of a straight line which is parallel to $y = 3x + 7$ and passes through point (2, -5).

Ans:

$y = mx + c$

$y = 3x + c$ because the gradients of parallel line are the same.

$-5 = 3 \times 2 + c$

$-5 = 6 + c$

$c = -5 - 6 = -11$

$y = 3x - 11$ is the equation of the new line

Example: Find the equation of a straight line which is parallel to $2y = 3x + 7$ and passes through point (2, -5).

Ans:

$2y = 3x + 7$

$y = \frac{3}{2}x + \frac{7}{2}$

$y = mx + c$

$y = \frac{3}{2}x + c$ Equation of the new line.

$-5 = \frac{3}{2} \times 2 + c$

$-5 = 3 + c$

$c = -5 - 3 = -8$

$y = \frac{3}{2}x - 8$ is the equation of the new line

Example: Find the equation of a straight line which is parallel to $2y = \frac{3}{4}x + 7$ and passes through point (-2, 5).

Ans:

$2y = \frac{3}{4}x + 7$

$y = \frac{3}{8}x + \frac{7}{2}$

$y = mx + c$

$y = \frac{3}{8}x + c$ Equation of the new line.

$5 = \frac{3}{8} \times (-2) + c$

$5 = -\frac{6}{8} + c$

$c = 5 + \frac{6}{8} = \frac{40}{8} + \frac{6}{8} = \frac{46}{8}$

$y = \frac{3}{8}x + \frac{46}{8}$ is the equation of the new line

Example: Find the equation of a straight line which is perpendicular to $y = 3x + 7$ and passes through point (2, -5).

Ans:

$$y = 3x + 7$$
$$y = mx + c$$
$$m = -\frac{1}{3}$$ Gradient of the new line.

$$y = -\frac{1}{3}x + c$$
$$-5 = -\frac{1}{3} \times 2 + c$$
$$-5 = -\frac{2}{3} + c$$
$$c = -5 + \frac{2}{3} = \frac{-15}{3} + \frac{2}{3} = \frac{-15+2}{3} = -\frac{13}{3}$$
$$y = -\frac{1}{3}x - \frac{13}{3}$$ is the equation of the new line

Example: Find the equation of a straight line which is perpendicular to $2y = 3x + 7$ and passes through point (-2, 3).

Ans:

$$y = mx + c$$

$$2y = 3x + 7$$
$$y = \frac{3}{2}x + \frac{7}{2}$$
$$m = -\frac{2}{3}$$ Gradient of the new line.
$$y = mx + c$$
$$y = -\frac{2}{3}x + c$$
$$3 = -\frac{2}{3} \times (-2) + c$$
$$3 = \frac{4}{3} + c$$
$$c = 3 - \frac{4}{3} = \frac{9}{3} - \frac{4}{3} = \frac{9-4}{3} = \frac{5}{3}$$
$$y = -\frac{2}{3}x + \frac{5}{3}$$ is the equation of the new line

Example: Find the equation of a straight line which is perpendicular to $2y = \frac{3}{4}x + 7$ and passes through point (-2, 5).

Ans:

$$2y = \frac{3}{4}x + 7$$

$$y = \frac{3}{8}x + \frac{7}{2}$$

$$m = -\frac{8}{3}$$ Gradient of the new line.

$$y = mx + c$$

$$y = -\frac{8}{3}x + c$$

$$5 = -\frac{8}{3} \times (-2) + c$$

$$5 = \frac{16}{3} + c$$

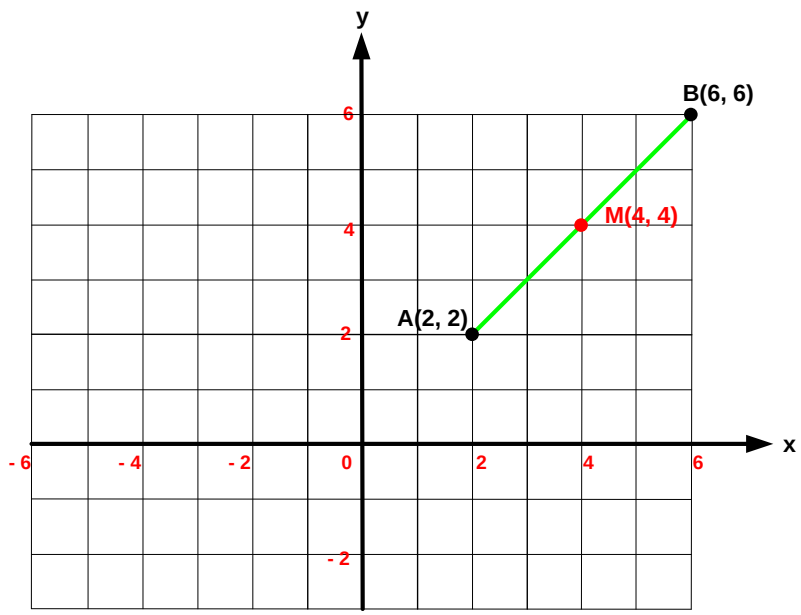
$$c = 5 - \frac{16}{3} = \frac{15}{3} - \frac{16}{3} = \frac{15-16}{3} = -\frac{1}{3}$$

$$y = -\frac{8}{3}x - \frac{1}{3}$$ is the equation of the new line

Midpoints of coordinate pairs:

The x-coordinate of Midpoint M $= \frac{x_A + x_B}{2} = \frac{6+2}{2} = \frac{8}{2} = 4$

The y-coordinate of Midpoint M $= \frac{y_A + y_B}{2} = \frac{6+2}{2} = \frac{8}{2} = 4$



Example: Find the coordinates of the midpoint of the line segment shown in Fig.6

The midpoint M of the line segment joining $A(x_A, y_A)$ and $B(x_B, y_B)$ is given by

Midpoint M $= \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right) = \left(\frac{-5+3}{2}, \frac{-2+4}{2}\right) = (-1, 1)$

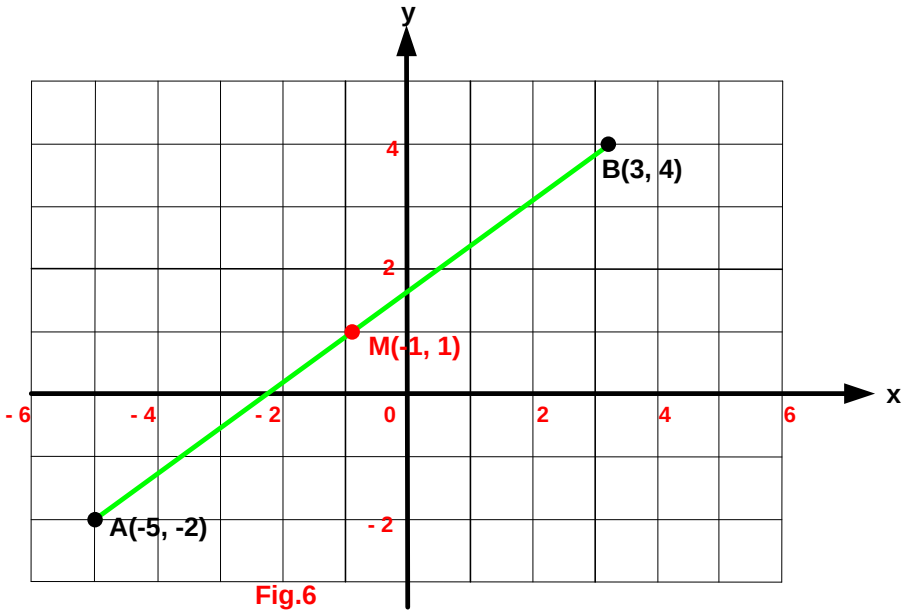


Fig.6

Example: Find the coordinates of the midpoint of the line segment shown in Fig.7

Midpoint M $= \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right) = \left(\frac{-5+5}{2}, \frac{5-2}{2}\right) = (0, 1.5)$

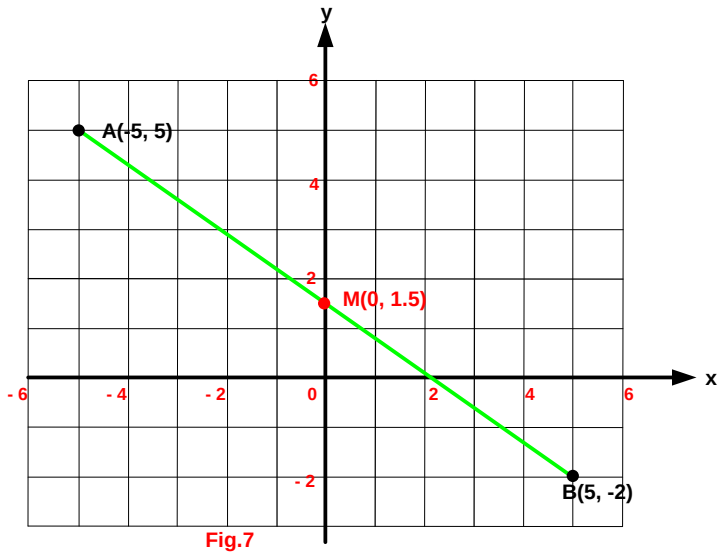


Fig.7

Example: **A line segment AB has midpoint M. Given A and M find the coordinates of B.**

(a) A(2, 2) and M(3, 4)
(b) A(2.3, -1.3) and M(-3.4, -2.5)

Ans:

(a) The midpoint M of the line segment joining $A(x_A, y_A)$ and $B(x_B, y_B)$ is given by Midpoint M is: $=\left(\frac{x_A+x_B}{2}, \frac{y_A+y_B}{2}\right)=\left(\frac{2+x_B}{2}, \frac{2+y_B}{2}\right)=(3,4)$

$$\frac{2+x_B}{2}=3 \Rightarrow 2+x_B=6 \Rightarrow x_B=6-2=4$$

$$\frac{2+y_B}{2}=4 \Rightarrow 2+y_B=8 \Rightarrow y_B=8-2=6$$

Therefore, point B = (4, 6)

(b) The midpoint M of the line segment joining $A(x_A, y_A)$ and $B(x_B, y_B)$ is given by Midpoint M is: $=\left(\frac{x_A+x_B}{2}, \frac{y_A+y_B}{2}\right)=\left(\frac{2+x_B}{2}, \frac{2+y_B}{2}\right)=(-3.4,-2.5)$

$$\frac{2.3+x_B}{2}=-3.4 \Rightarrow 2.3+x_B=-6.8 \Rightarrow x_B=-6.8-2.3=-9.1$$

$$\frac{-1.3+y_B}{2}=-2.5 \Rightarrow -1.3+y_B=-5 \Rightarrow y_B=-5+1.3=-3.7$$

Therefore, point B = (-9.1, -3.7)

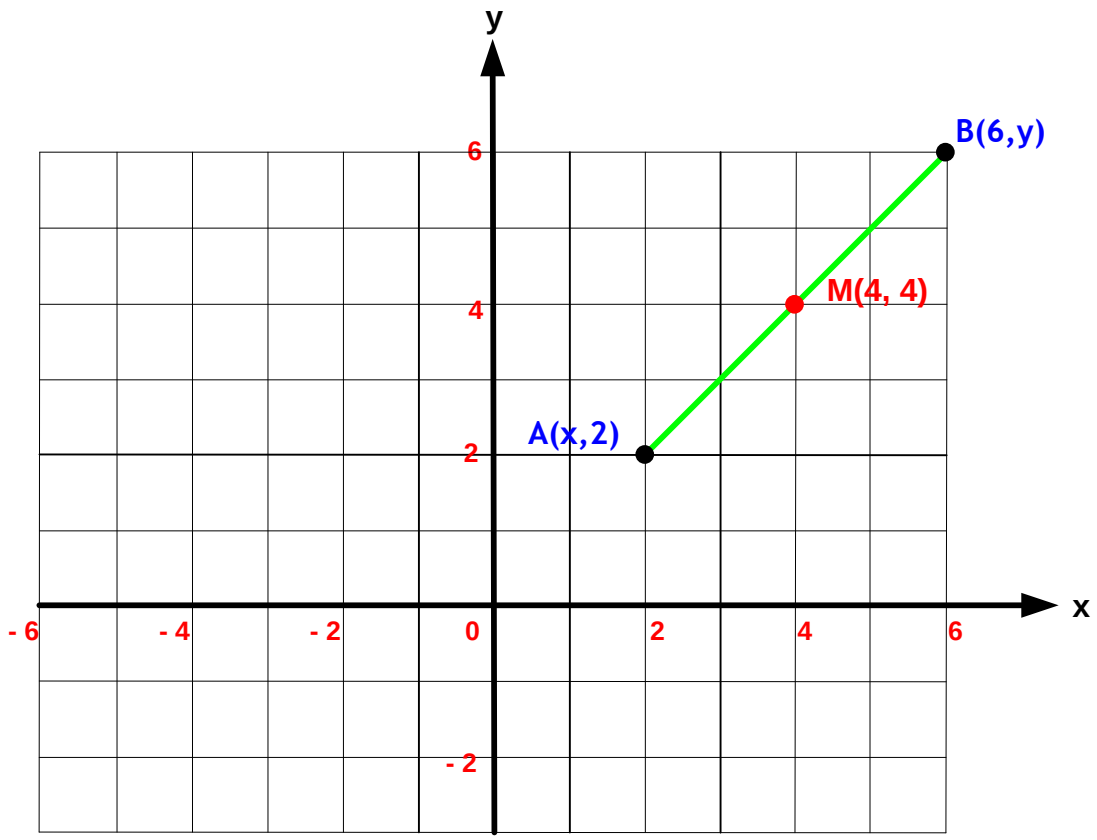
Example: **A line segment AB has midpoint M. Given B and M find the coordinates of A. Given: B(2, 2) and M(3, 4)**

The midpoint M of the line segment joining $A(x_A, y_A)$ and $B(x_B, y_B)$ is given by Midpoint M is: $=\left(\frac{x_A+x_B}{2}, \frac{y_A+y_B}{2}\right)=\left(\frac{2+x_B}{2}, \frac{2+y_B}{2}\right)=(3,4)$

$$\frac{x_A+2}{2}=3 \Rightarrow x_A+2=6 \Rightarrow x_A=6-2=4$$

$\frac{y_A+2}{2}=4 \Rightarrow y_A+2=8 \Rightarrow y_A=8-2=6$. **Therefore, point A = (4, 6).**

Example: A line segment AB has point A(x,2) and point B(6,y). The coordinates of a midpoint is: M (4,4). Find the values of x and y.



Ans: The midpoint M of the line segment joining $A(x_A, y_A)$ and $B(x_B, y_B)$ is given by Midpoint M is: $=\left(\frac{x_A+x_B}{2}, \frac{y_A+y_B}{2}\right)$

$$\frac{x+6}{2}=4$$

$$x+6=8$$

$$x=8-6$$

$$x=2$$

And $\frac{2+y}{2}=4$

$$2+y=8$$

$$y=8-2 \qquad y=6$$

Length of a Straight Line:

The length of a straight line between two points $p_1(x_1, y_1)$ **and** $p_2(x_2, y_2)$ **is give by:** $p_1p_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. For example in the case of Fig.8, the length of straight line p_1p_2 is:

$$p_1p_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6-2)^2 + (5-2)^2} = \sqrt{(4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

OR using **Pythagoras Theorem**, because the triangle $p_1p_2p_3$ is a **right angle triangle:**

$$p_1p_2 = \sqrt{(p_1p_3)^2 + (p_3p_2)^2} = \sqrt{(4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

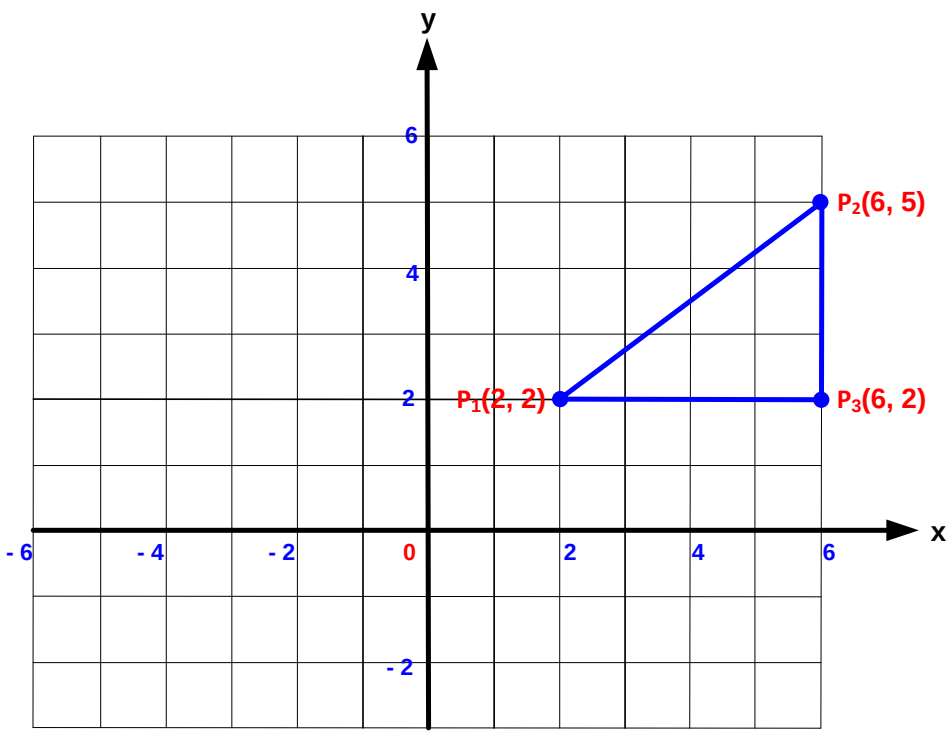


Fig.8

Example: **Find the lenght of a straight line which passes through points A(6, 2) and B(-3, -2).**

Ans:

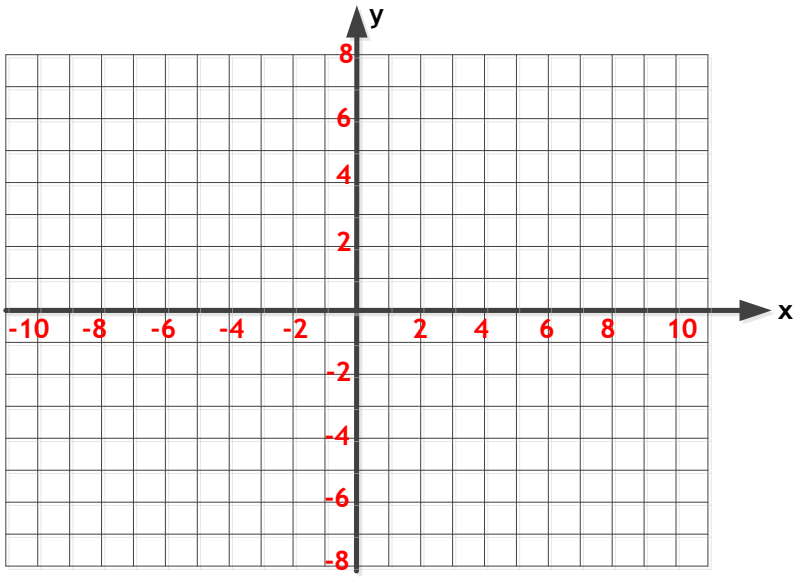
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-2-2)^2 + (-3-6)^2} = \sqrt{(-4)^2 + (-9)^2} = \sqrt{16+81} = \sqrt{97}$$

Exercises

1. Plot the following points on the graph below.

$P_1(2,-2)$ $P_2(8,2)$ $P_3(-3,0)$ $P_4(-9,6)$

$P_5(0,3)$ $P_6(3,7)$ $P_7(0,-3)$ $P_8(-5,-7)$



2. Plot the graphs of the following straight lines.

(a) $y = x - 1$ (b) $2y = 4 - \frac{2}{3}x$

3. Plot the graphs of the following straight lines.

(a) $y = 5$ (b) $y = -3$ (c) $x = -3$

4. Find the equation of a straight line which passes through points P_1 and P_2 as shown in Fig.1

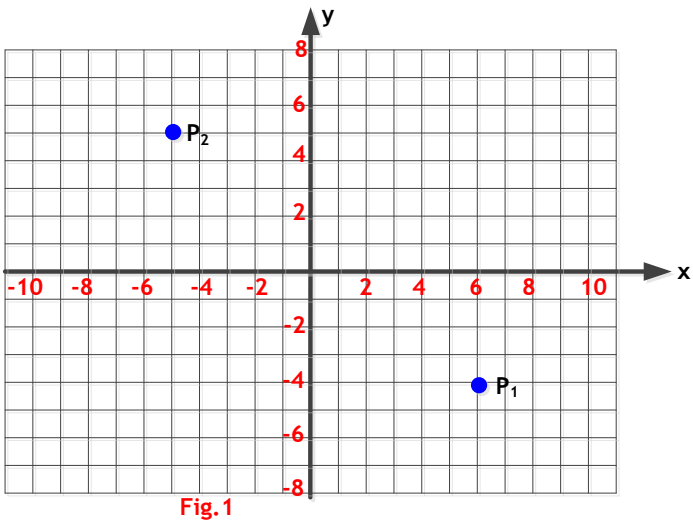
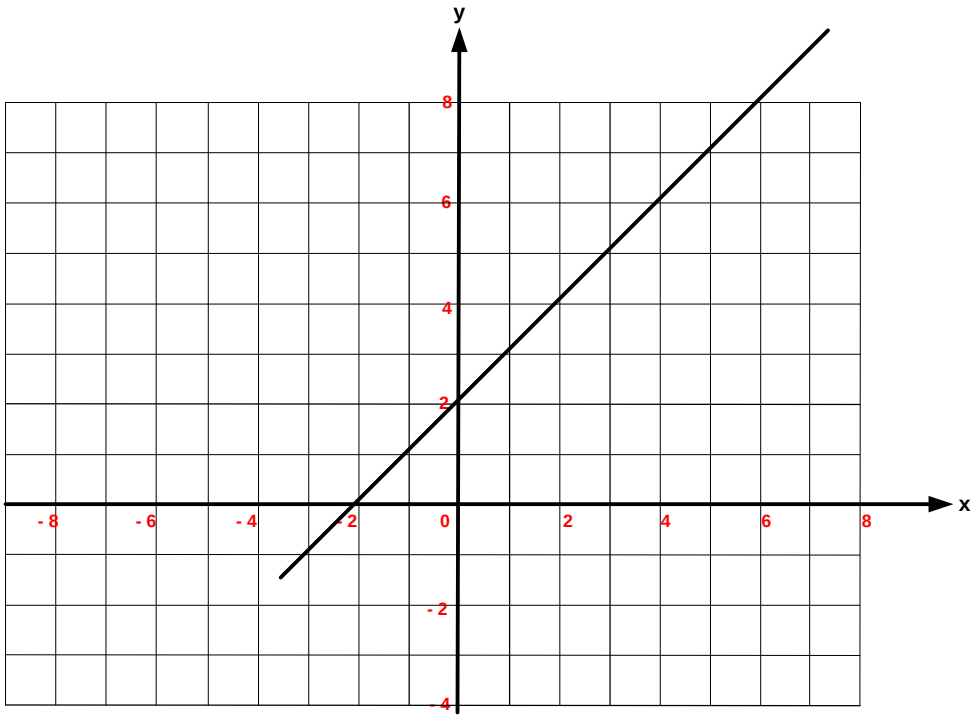


Fig.1

5. Find the equation of a straight line as shown below.



6. Copy and complete the table for each of these equations.

Equation	Gradient	Coordinates of y-intercept
$y = 3x - 5$		
$2y = 4 - 3x$		
$\frac{2}{3}y = 4 - \frac{3}{5}x$		

7. Write the equations of these FIVE straight lines.

(a) Line **A** has a gradient of **-5** and y-intercept at **(0, -4).**

(b) Line **B** cuts the y-axis at **-4** and has a gradient of **3**.

(c) Line **C** has **m = 5** and $c = \frac{1}{3}$

(d) Line **D** cuts the x-axis at **-4** and has gradient of **2**.

(e) Line **E** passes through the origin and has gradient of -5.

8. Find the gradient and the y-intercept of $3y - 4x = 5$.

9. Find the gradient and the y-intercept of $-x-5=3y$.

10. Find the gradient and the y-intercept of $\frac{2}{5}y-\frac{3}{7}x-7=0$.

11. Find the equation of a line with gradient of -2 and passes through point A(1, 3) .

12. Find the gradient and the y-intercept of the following straight lines.
(a) $3y-4x=12$
(b) $\frac{x}{4}+\frac{y}{3}=5$
(c) $\frac{x}{4}-\frac{y}{3}=2$

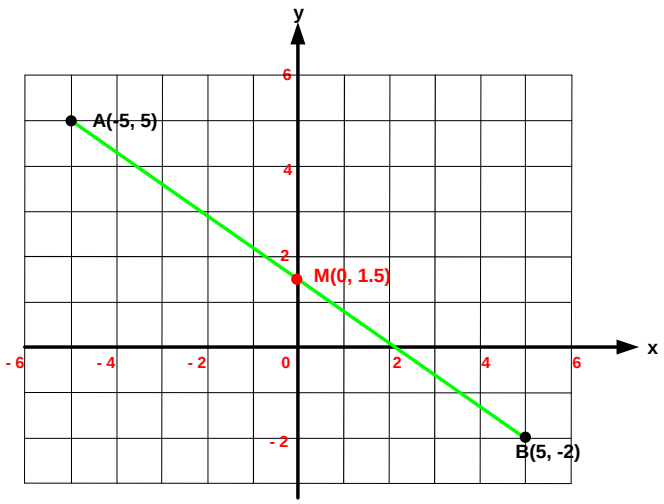
13. Find the equation of a straight line which is parallel to $2y=5x-7$ and passes through point (2, -3).

14. Find the equation of a straight line which is parallel to $\frac{x}{3}+\frac{y}{5}=2$ and passes through point (3, 5).

15. Find the equation of a straight line which is perpendicular to $3y=-2x-5$ and passes through point (-1, 5).

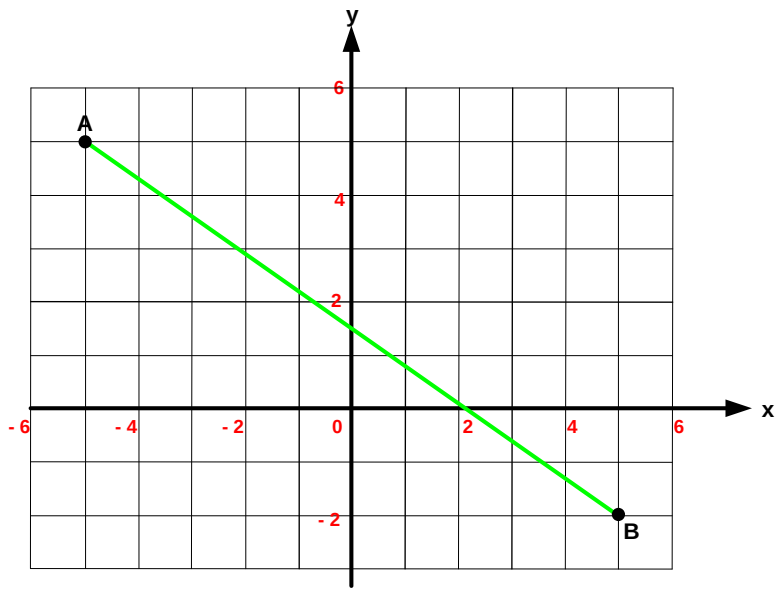
16. Find the equation of a straight line which is perpendicular to $\frac{3x}{4}-\frac{2y}{5}=7$ and passes through point (4, -10).

17. Find the coordinates of the midpoint of the line segment AB shown below.



18. A line segment AB has midpoint M. Given B and M find the coordinates of A. Given: B(2, 2) and M(3, 4)

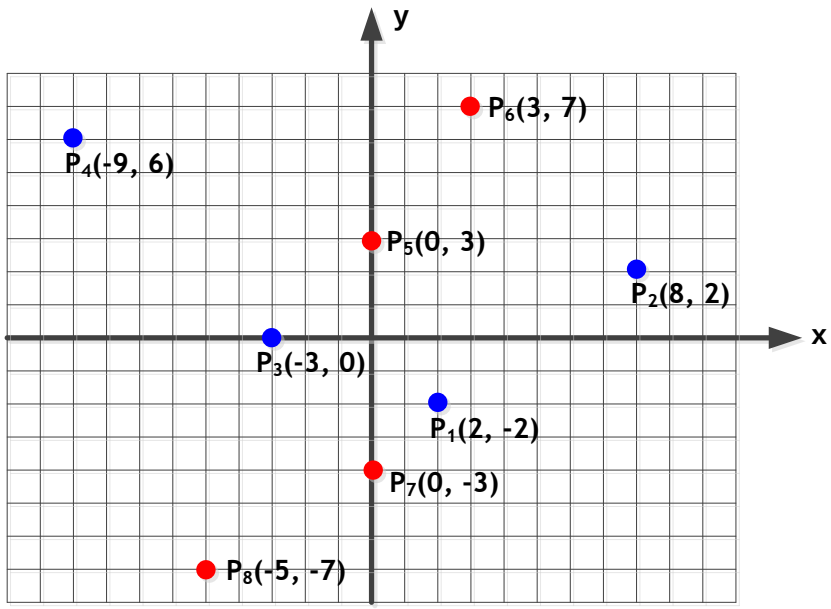
19. Calculate the length of straight line AB.



Solutions to the Exercises

1. Plot the following points on the graph below.

$P_1(2,-2)$ $P_2(8,2)$ $P_3(-3,0)$ $P_4(-9,6)$
$P_5(0,3)$ $P_6(3,7)$ $P_7(0,-3)$ $P_8(-5,-7)$



2. Plot the graphs of the following straight lines.

(a) $y = x - 1$ (b) $2y = 4 - \frac{2}{3}x$

Ans:

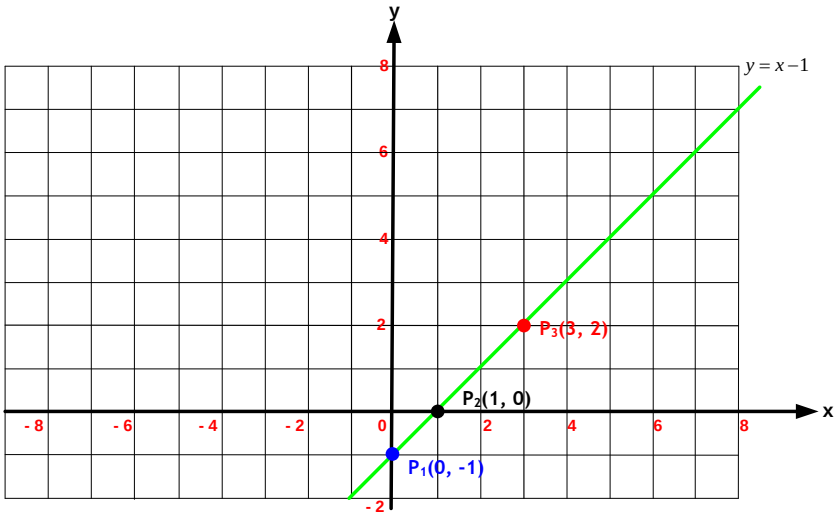
(a) $y = x - 1$

x	y
0	-1
1	0

OR

x	y
0	- 1
3	2

$P_1(0, -1)$ **and** $P_2(1, 0)$ **OR** $P_1(0, -1)$ **and** $P_3(3, 2)$



(b) $2y = 4 - \frac{2}{3}x$

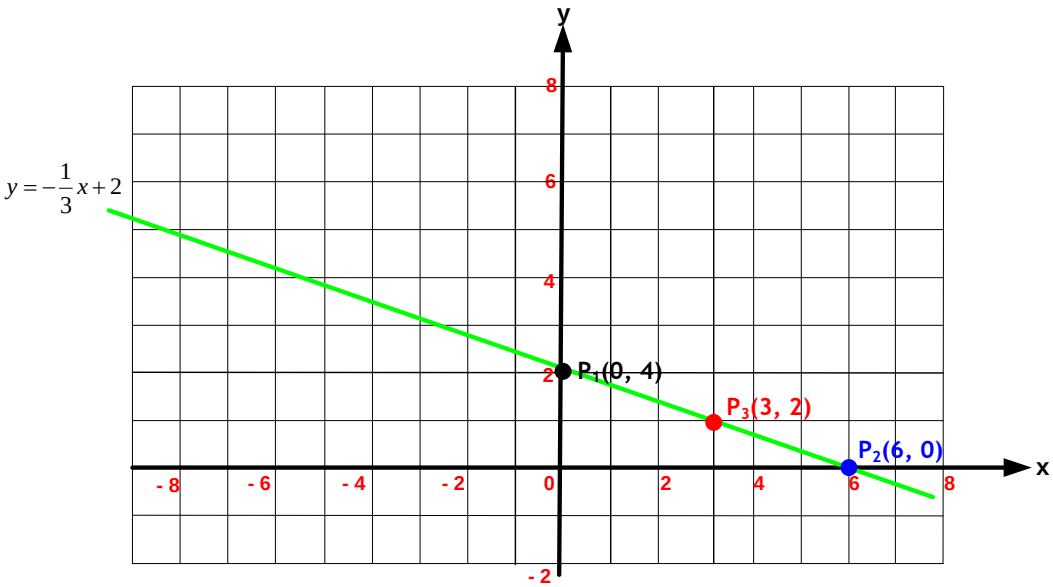
$y = -\frac{1}{3}x + 2$

x	y
0	2
6	0

OR

x	y
0	2
3	1

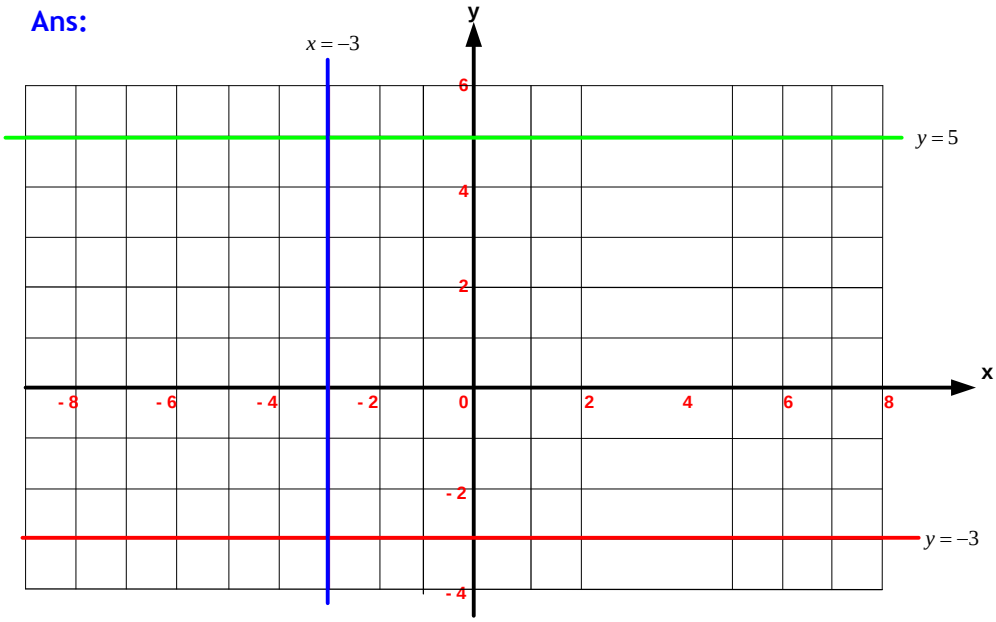
$P_1(0, 2)$ **and** $P_2(6, 0)$ **OR** $P_1(0, 2)$ **and** $P_3(3, 1)$



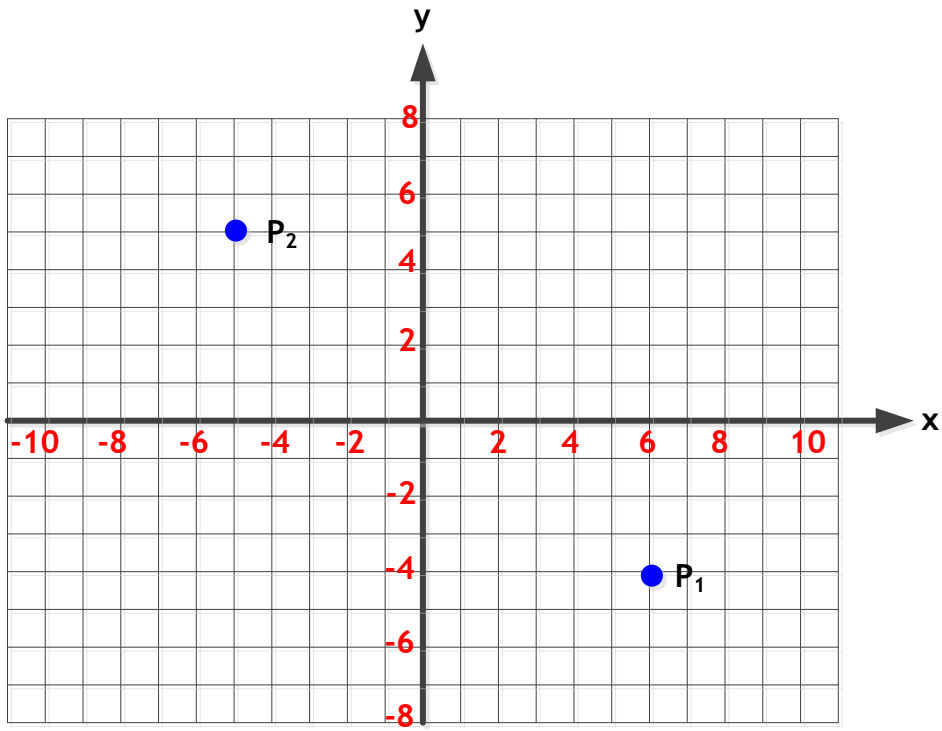
3. Plot the graphs of the following straight lines.

(a) $y=5$ (b) $y=-3$ (c) $x=-3$

Ans:



4. Find the equation of a straight line which passes through points P_1 and P_2 as shown below.



Ans:

The standard equation of a straight line is: $y=mx+c$, so we need to find the gradient, m and the y-intercept c as follows:

$P_1(6,-4)$ and $P_2(-5,5)$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{5-(-4)}{-5-6}=\frac{5+4}{-11}=-\frac{9}{11}$$

$$y=mx+b$$

$$y=-\frac{9}{11}x+c$$

$$5=-\frac{9}{11}(-5)+c$$

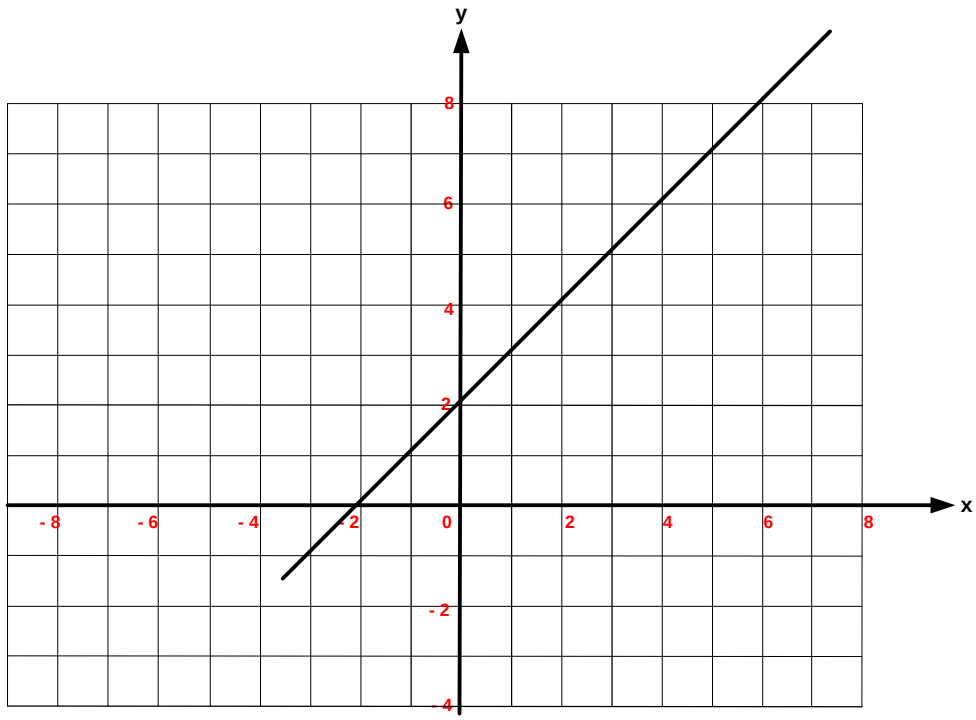
$$5=\frac{45}{11}+c$$

$$c=5-\frac{45}{11}$$

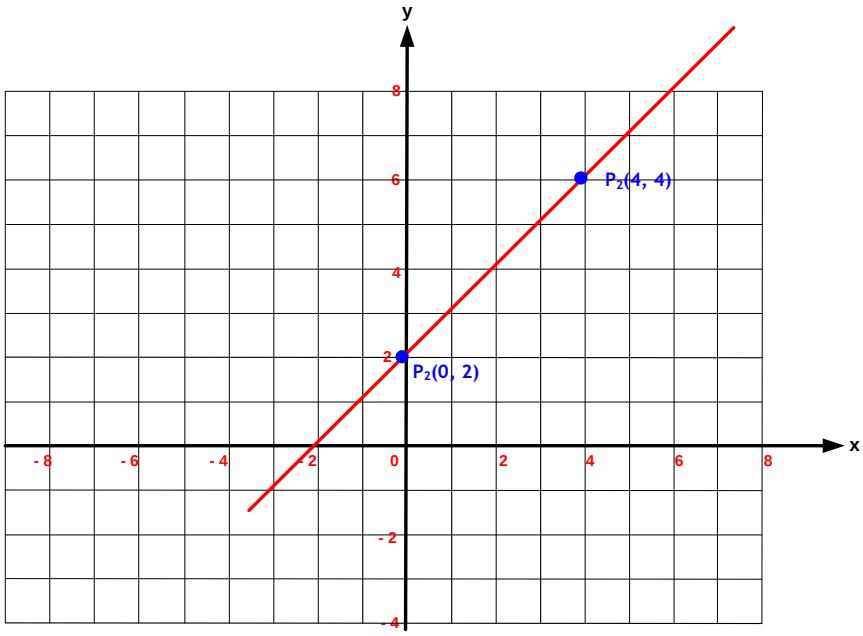
$$c=\frac{55-45}{11}=\frac{10}{11}$$

$$y=-\frac{9}{11}x+\frac{10}{11}$$

5. Find the equation of a straight line as shown below.



Ans:



Looking at the graph, the line crosses the y axis at (0, 2), and therefore, the y-intercept ***c*** = 2. The gradient is positive and it is: $m=\frac{y_2-y_1}{x_2-x_1}=\frac{4-2}{4-0}=\frac{2}{4}=\frac{1}{2}$

$y=mx+c$

$y=\frac{1}{2}x+2$

6. Copy and complete the table for each of these equations.

Equation	Gradient	Coordinates of y-intercept
$y=3x-5$		
$2y=4-3x$		
$\frac{2}{3}y=4-\frac{3}{5}x$		

(a) $y=3x-5$ (b) $2y=4-3x$ (c) $\frac{2}{3}y=4-\frac{3}{5}x$

Ans:

(a) $y=3x-5$

$y=mx+c$ **Equation of a straight line**

Hence, $m=gradient=3$ and $y-\text{intercept}=c=-5$

(b) $2y = 4 - 3x$

$2y = -3x + 4$

$y = -\frac{3}{2}x + 2$

$y = mx + c$ Equation of a straight line

Hence, $m = gradient = -\frac{3}{2}$ and $y - intercept = c = 2$

(c) $\frac{2}{3}y = 4 - \frac{3}{5}x$

$2y = 12 - \frac{9}{5}x$

$2y = -\frac{9}{5}x + 12$

$y = -\frac{9}{10}x + 6$

$y = mx + c$ Equation of a straight line

Hence, $m = gradient = -\frac{9}{10}$ and $y - intercept = c = 6$

Equation	Gradient	Coordinates of y-intercept
$y = 3x - 5$	3	-5
$2y = 4 - 3x$	$-\frac{3}{2}$	2
$\frac{2}{3}y = 4 - \frac{3}{5}x$	$-\frac{9}{10}$	6

7. Write the equations of these FIVE straight lines.

(a) Line **A** has a gradient of **-5** and y-intercept at **(0, -4).**

Ans: $y = mx + c$

$m = -5$

$c = -4$

$y = -5x - 4$

(b) Line **B** cuts the y-axis at **-4** and has a gradient of **3**.

Ans: $y = mx + c$

$m = 3$

$c = -4$

$y = 3x - 4$

(c) Line **C** has **m = 5** and $c = \frac{1}{3}$

Ans: $y = mx + c$

$m = 5 \quad c = \frac{1}{3} \Rightarrow y = 5x + \frac{1}{3}$

(d) Line D cuts the x-axis at -4 and has gradient of 2.

Ans: $y = mx + c$

$m = 2$

$y = 2x + c$

$0 = 2 \times (-4) + c$

$c = 8$

$y = 2x + 8$

(e) Line E passes through the origin and has gradient of -5.

Ans: $y = mx$

$y = -5x$

8. Find the gradient and the y-intercept of $3y - 4x = 5$.

Ans:

$3y - 4x = 5$

$3y = 4x + 5$

$y = \frac{4}{3}x + \frac{5}{3}$

Gradient $= m = \frac{4}{3}$ **and y-intercept** $= c = \frac{5}{3}$

9. Find the gradient and the y-intercept of $-x - 5 = 3y$.

Ans:

$-x - 5 = 3y$

$3y = x + 5$

$y = \frac{1}{3}x + \frac{5}{3}$

Gradient $= m = \frac{1}{3}$ **and y-intercept** $= c = \frac{5}{3}$

10. Find the gradient and the y-intercept of $\frac{2}{5}y - \frac{3}{7}x - 7 = 0$.

Ans:

$\frac{2}{5}y - \frac{3}{7}x - 7 = 0$

$\frac{2}{5}y = \frac{3}{7}x + 7$

$\frac{5}{2}\left(\frac{2}{5}y\right) = \left(\frac{5}{2}\right)\left(\frac{3}{7}x + 7\right)$

$y = \frac{15}{14}x + \frac{35}{2}$

Gradient $= m = \frac{15}{14}$ **and y-intercept** $= c = \frac{35}{2}$

11. **Find the equation of a line with gradient of -2 and passes through point A(1, 3) .**

Ans:

$$y = mx + c$$
$$y = -2x + c$$
$$3 = -2 \times 1 + c$$
$$c = 5$$
$$y = -2x + 5$$

12. **Find the gradient and the y-intercept of the following straight lines.**

(a) $3y - 4x = 12$

Ans:

$$y = mx + c$$
$$3y = 4x + 12$$
$$y = \frac{4}{3}x + \frac{12}{3}$$
$$y = \frac{4}{3}x + 4$$
$$\Rightarrow m = \frac{4}{3} \quad and \quad c = 4$$

(b) $\frac{x}{4} + \frac{y}{3} = 5$

Ans:

$$y = mx + c$$
$$\frac{3x + 4y}{12} = 5$$
$$3x + 4y = 60$$
$$4y = -3x + 60$$
$$y = -\frac{3}{4}x + 15$$
$$\Rightarrow m = -\frac{3}{4} \quad and \quad c = 15$$

(c) $\frac{x}{4} - \frac{y}{3} = 2$

Ans:

$$y = mx + c$$
$$\frac{3x - 4y}{12} = 2$$
$$3x - 4y = 24 \quad \Rightarrow -4y = -3x + 24 \quad \Rightarrow y = \frac{-3}{-4}x + \frac{24}{-4}$$
$$y = \frac{3}{4}x - 6 \quad \Rightarrow m = \frac{3}{4} \quad and \quad c = -6$$

13. Find the equation of a straight line which is parallel to $2y=5x-7$ and passes through point (2, -3).

Ans:

$$2y=5x-7$$
$$y=\frac{5}{2}x-\frac{7}{2}$$
$$y=mx+c$$
$$y=\frac{5}{2}x+c$$ Equation of the new line.
$$-3=\frac{3}{2}\times 2+c$$
$$-3=3+c \qquad \Rightarrow c=-3-3=-6$$
$$y=\frac{5}{2}x-6$$ Equation of that line.

14. Find the equation of a straight line which is parallel to $\frac{x}{3}+\frac{y}{5}=2$ and passes through point (3, 5).

Ans:

$$\frac{x}{3}+\frac{y}{5}=2$$
$$\frac{5x+3y}{15}=2$$
$$5x+3y=30$$
$$3y=-5x+30$$
$$y=-\frac{5}{3}x+\frac{30}{3}$$
$$y=-\frac{5}{3}x+10$$
$$y=mx+c$$
$$m=-\frac{5}{3}$$ of the new line. Because they are parallel.
$$y=-\frac{5}{3}x+c$$ Equation of the new line.
$$5=-\frac{5}{3}\times 3+c \qquad \Rightarrow 5=-5+c \quad \Rightarrow c=5+5=10 \quad \Rightarrow y=-\frac{5}{3}x+10$$
Equation of that line.

15. Find the equation of a straight line which is perpendicular to $3y=-2x-5$ and passes through point (-1, 5).

Ans:

$$y=mx+c$$
$$3y=-2x-5$$
$$y=-\frac{2}{3}x-\frac{5}{3}$$

$m=-\left(-\frac{3}{2}\right)=\frac{3}{2}$ Gradient of the new line.

$y=mx+c$

$y=\frac{3}{2}x+c$

$5=\frac{3}{2}\times(-1)+c$

$3=-\frac{3}{2}+c$

$c=3+\frac{3}{2}=\frac{6+3}{2}=\frac{9}{2}$

$y=\frac{3}{2}x+\frac{9}{2}$ Equation of that line.

16. Find the equation of a straight line which is perpendicular to $\frac{3x}{4}-\frac{2y}{5}=7$ and passes through point (4, -10).

Ans:

$\frac{3x}{4}-\frac{2y}{5}=7$

$\frac{15x-8y}{20}=7$

$15x-8y=140$

$-8y=-15x+140$

$y=\frac{-15}{-8}x-\frac{140}{8}$

$y=\frac{15}{8}x-\frac{35}{2}$

$y=mx+c$

$m=-\frac{8}{15}$ of the new line. Because they are perpendicular.

$y=-\frac{8}{15}x+c$ Equation of the new line.

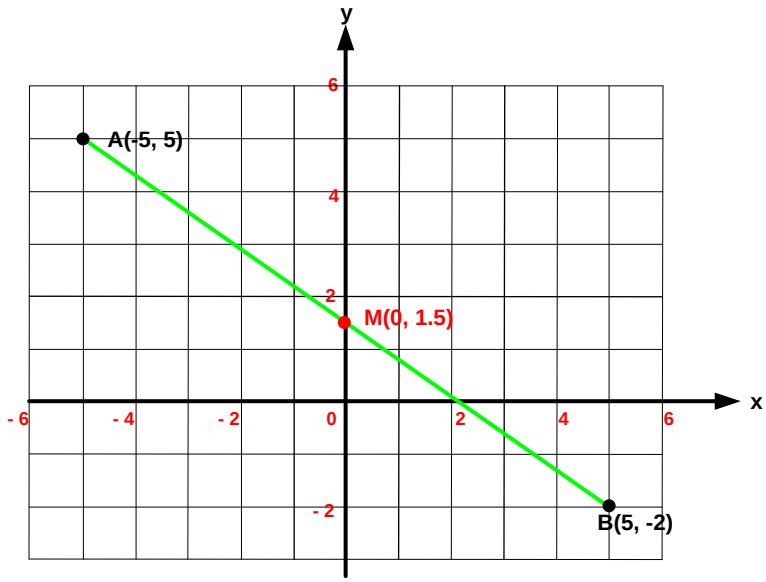
$-10=-\frac{8}{15}\times 4+c$

$-10=-\frac{32}{15}+c$

$c=\frac{32}{15}-10 \quad \Rightarrow c=\frac{32-150}{15}=-\frac{118}{15} \quad \Rightarrow y=-\frac{8}{15}x-\frac{118}{15}$

Equation of that line.

17. **Find the coordinates of the midpoint of the line segment AB shown below.**



Midpoint M $= \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right) = \left(\frac{-5+5}{2}, \frac{5-2}{2}\right) = (0, 1.5)$

18. **A line segment AB has midpoint M. Given B and M find the coordinates of A. Given: B(2, 2) and M(3, 4)**

The midpoint **M** of the line segment joining $A(x_A, y_A)$ and $B(x_B, y_B)$

is given by Midpoint **M** is: $= \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right) = \left(\frac{2 + x_B}{2}, \frac{2 + y_B}{2}\right) = (3, 4)$

$$\frac{x_A + 2}{2} = 3 \Rightarrow x_A + 2 = 6 \Rightarrow x_A = 6 - 2 = 4$$

$$\frac{y_A + 2}{2} = 4 \Rightarrow y_A + 2 = 8 \Rightarrow y_A = 8 - 2 = 6.$$ **Therefore, point A = (4, 6).**

19. Calculate the length of straight line AB.

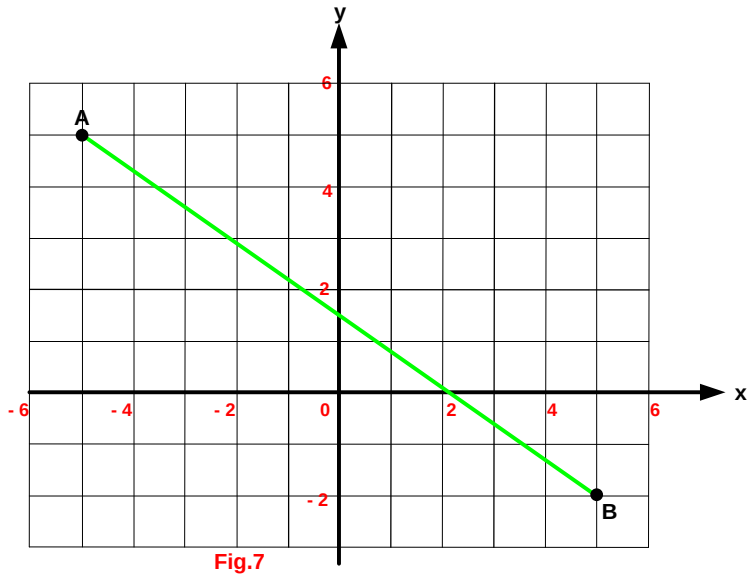


Fig.7

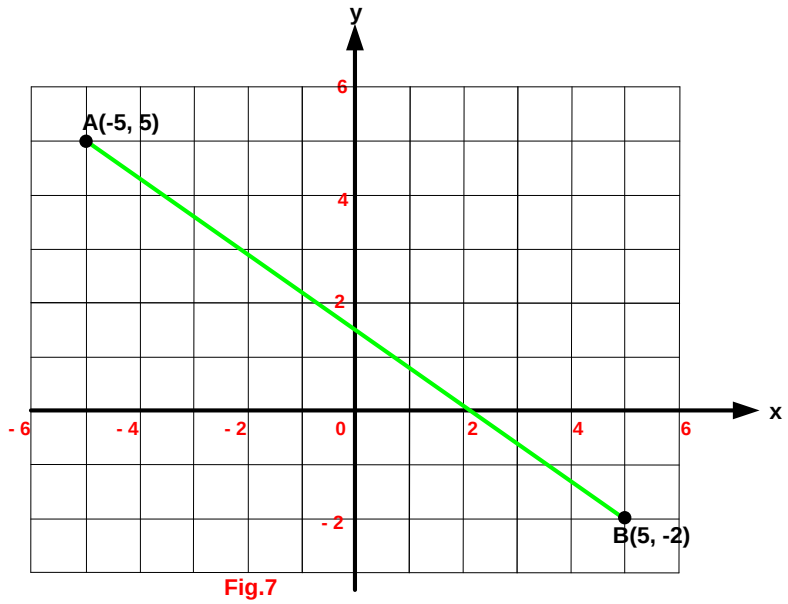


Fig.7

$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(5-(-5))^2+(-2-5)^2}=\sqrt{(10)^2+(-7)^2}=\sqrt{100+49}=\sqrt{149}$$

الله(ج) د وکړي چې تاسې ما ته، زما مور او پلار ته، زما اولادونو ته، زما ټولې کورنۍ اوټولو مسلمانانو ته د زړه له کومې دعا کړيې وي، **اوکنه؟** نو اوس يې لطفاً وکړئ!

په پای کې له تاسو ټولو سره له لرې ځايه خدای په اماني کوم. له الله(ج) څخه هيله کوم چې په افغانستان او په ټوله نړۍ کې هغه شانته صلح راشي، چې الله(ج) ته مقبوله او منظوره وي، او الله(ج) دې، د افغانانو ترمنځ د حق فيصله وکړي. آمين يارب العٰلمين.